Chapter 1: Introduction

Dep. of Information and Communications Eng.
Jaeshin Jang
(icjoseph@inje.ac.kr)

What is a Signal?

- Type of signals
  - Speech signal, visual signal, e-mail over internet etc.
  - Heartbeat, blood pressure, temperature of a patient
  - Weather forecast, stock price

A *signal* is formally defined as a function of one or more variables that conveys *information* on the nature of a physical phenomenon.

- One dimensional vs. multidimensional signals
  - Speech signal → One dimensional
  - Image signal → Multidimensional

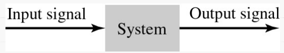
What is System?

A *system* is an entity that manipulates one or more signals *to accomplish a function*, thereby yielding new signals.

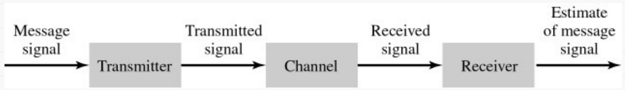
- Examples of system
  - Vocal track, electronic systems
- The purpose of system depends on the application
  - Automatic speaker recognition system : recognizing the speaker
  - Communication system : transporting the information
  - Aircraft landing system : To keep the aircraft on the extended centerline of a runway

Overview of Specific Systems

- Block diagram of a system



- Communication systems



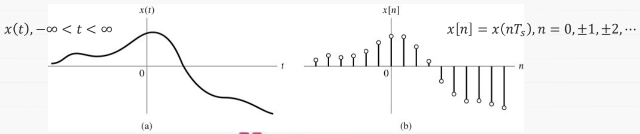
- Sampling
- Quantization
- Coding

- Channel coding/decoding

- Broadcasting
- Point-to-point communication

Classification of Signals (1)

- In this book,
  - **Single** valued signal or **one-dimensional** signal
  - **Real**-valued signal or **complex**-valued signal
- (1) Continuous-time vs. discrete-time signals
  - Continuous-time signal if it is defined for all time t
  - Discrete-time signal if it is defined only at discrete instants of time
    - Continuous-time signal → Discrete-time signal : sampling



Classification of Signals (2)

- (2) Even and odd signals
  - Even signal if $x(-t) = x(t)$ for all t → symmetric
  - Odd signal if $x(-t) = -x(t)$ for all t → anti-symmetric

Example 1.1 EVEN AND ODD SIGNALS

$$x(t) = \begin{cases} \sin\left(\dfrac{\pi t}{T}\right), & -T \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

Is $x(t)$ odd or even function ?

Classification of Signals (3)

- In general $x(t) = x_e(t) + x_o(t)$, therefore

$$\boxed{x_e(t) = \frac{1}{2}[x(t) + x(-t)];} \quad \boxed{x_o(t) = \frac{1}{2}[x(t) - x(-t)]}$$

Example 1.2 EVEN AND ODD SIGNALS

$x(t) = e^{-2t}\cos t$ Find the even and odd components of the signal.

- *Conjugate symmetric* if (Problem 1.1, 1.2)

$$\boxed{x(-t) = x^*(t)} \iff a(t) + jb(t) \Leftrightarrow a(t) - jb(t) \Rightarrow \begin{cases} a(t) = a(-t) \\ b(-t) = -b(t) \end{cases}$$

Classification of Signals (4)

- (3) Periodic signals vs. non-periodic signal

  A *periodic signal* if it satisfies the condition,

$$\boxed{x(t) = x(t+T), \quad \text{for all } t} \qquad (1.7)$$

Fundamental period : smallest value of T that satisfies Eq. (1.7)

Fundamental frequency : the reciprocal of the fundamental period

$$f = \frac{1}{T} \quad \text{hertz (Hz) or cycles per second}$$

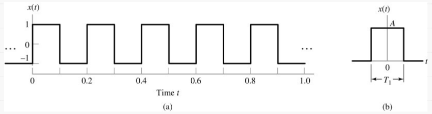
Angular frequency : measured in radians per second

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{[radian/sec]}$$

aperiodic signal of which no value of T satisfies Eq. (1.7)

(aperiodic or nonperiodic signal)

Classification of Signals (5)



periodic signal with amplitude *A=1*, and periodic *T=0.2s*

nonperiodic signal with amplitude *A*, and duration T_1

(Problem 1.3)

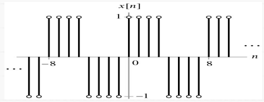
For discrete-time signal, *x[n]* is said to be periodic if

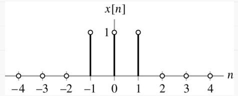
$$\boxed{x[n] = x[n+N], \quad \text{for integer } n}$$

Fundamental angular frequency of *x[n]* is defined by

$$\Omega = \frac{2\pi}{N} \quad \text{[radian]}$$

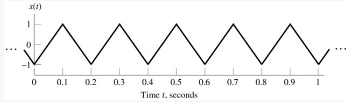
Classification of Signals (6)





Angular frequency?

(Problem 1.4, 1.5)



(4) Deterministic signals vs. random signals

Deterministic signal : there is no uncertainty with respect to its value at any time (ex. periodic signal)

Random signal : there is uncertainty before it occurs (ex. noise)

Classification of Signals (7)

(5) Energy signals and power signals

Instantaneous power dissipated in a resistor

$$p(t) = \frac{v^2(t)}{R} = i^2(t)R$$

For 1-ohm resistor condition, $p(t) = x^2(t)$

Total energy of the continuous-time signal, $x(t)$

$$\boxed{E = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)\,dt = \int_{-\infty}^{\infty} x^2(t)\,dt}$$

Time-averaged or average power,

$$\boxed{P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt}$$

For a periodic signal,

$$\boxed{P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt}$$

Classification of Signals (8)

In case of a discrete-time signal, total energy becomes

$$\boxed{E = \sum_{n=-\infty}^{\infty} x^2[n]}$$

Average power is defined by

$$\boxed{P = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N} x^2[n]}$$

For a periodic signal,

$$\boxed{P = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n]}$$

Energy signal if and only if $\boxed{0 < E < \infty}$

Power signal if and only if $\boxed{0 < P < \infty}$

Comments

- Energy and power classifications are mutually exclusive
- Energy signal : zero time-averaged power
- Power signal : infinite energy

(Problem 1.6 ~ 1.9)

Basic Operations on Signals (1)

- Operation performed on **dependent variables**

  Amplitude scaling : amplifier

  $\boxed{y(t) = cx(t)}$ $\boxed{y[n] = cx[n]}$ where c is scaling factor

  Addition

  $\boxed{y(t) = x_1(t) + x_2(t)}$ $\boxed{y[n] = x_1[n] + x_2[n]}$

  Physical example : audio mixer $(x_1 : \text{music signal}, \quad x_2 : \text{voice signal})$

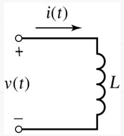
  Multiplication : AM radio signal

  $\boxed{y(t) = x_1(t)x_2(t)}$ $\boxed{y[n] = x_1[n]x_2[n]}$

  Differentiation

  $\boxed{y(t) = \frac{d}{dt}x(t)}$

  $v(t) = L\frac{d}{dt}i(t)$

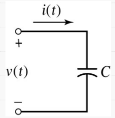


Basic Operations on Signals (2)

Integration

$\boxed{y(t) = \int_{-\infty}^{t} x(\tau)d\tau}$

$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$

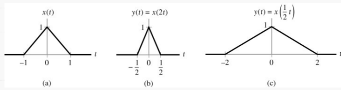


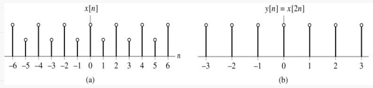
- Operation performed on **independent variable**

  Time scaling

  $\boxed{y(t) = x(at)}$ $\boxed{y[n] = x[kn], \quad k > 0}$

  - $a > 1$: compressed version of $x(t)$
  - $0 < a < 1$: expanded version of $x(t)$





(Problem 1.10)

Basic Operations on Signals (3)

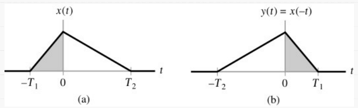
Reflection

$\boxed{y(t) = x(-t)}$

- Even signal : $x(-t) = x(t)$
- Odd signal : $x(-t) = -x(t)$

Example 1.3 REFLECTION

Find the reflected version of $x(t)$



(Problem 1.11, 1.12)

Time shifting

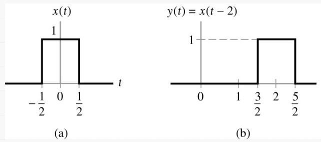
$\boxed{y(t) = x(t - t_0)}$ $\boxed{y[n] = x[n - m]}$

- $t_0 > 0$: y(t) is obtained by shifting x(t) toward right
- $t_0 < 0$: y(t) is obtained by shifting x(t) toward left

Basic Operations on Signals (4)

Example 1.4 TIME SHIFTING

Find $y(t) = x(t-2)$



(Problem 1.13)

- Precedence rule for time shifting and time scaling

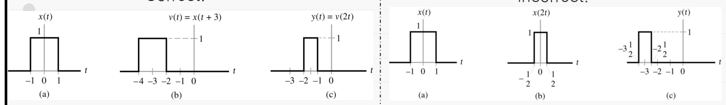
  $\boxed{y(t) = x(at - b)?} \Leftarrow \boxed{v(t) = x(t-b) \Rightarrow y(t) = v(at)}$

Example 1.5 TIME SHIFTING

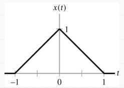
For $x(t)$ of unit amplitude and a duration of 2 time units, find $y(t) = x(2t+3)$

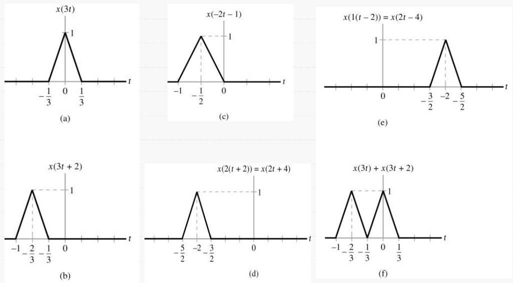
Basic Operations on Signals (5)

Correct! Incorrect!



Problem 1.14



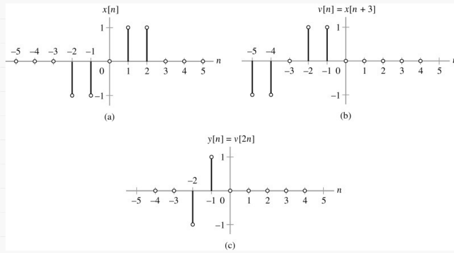


Basic Operations on Signals (6)

Example 1.6 Precedence rule for discrete-time signal

$$x[n]=\begin{cases}1, & n=1,2\\ -1, & n=-1,-2\\ 0, & n=0 \text{ and } |n|>2.\end{cases}$$

Find $y[n]=x[2n+3]$



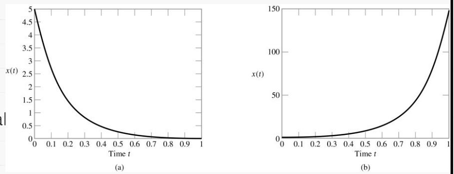
(Problem 1.15)

Elementary Signals (1)

Exponential Signals

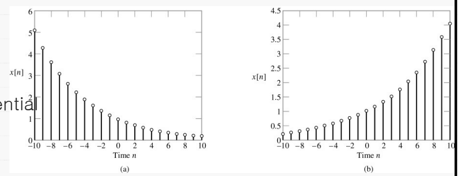
$x(t)=Be^{at}$

- $a>0$: Growing exponential
- $a<0$: Decaying exponential



$x[n]=Br^{n}$

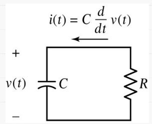
- $r>1$: Growing exponential
- $0<r<1$: Decaying exponential



Elementary Signals (2)

$$RC\frac{d}{dt}v(t)+v(t)=0$$

$$v(t)=V_0e^{-t/(RC)}$$



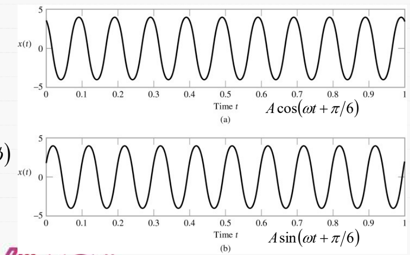
- *RC* : Time constant
- The larger the resistor *R*, the slower will be the rate of decay of $v(t)$

Sinusoidal Signals

$$x(t)=A\cos(\omega t+\phi)$$

Period : $T=\dfrac{2\pi}{\omega}$

$$\begin{aligned}x(t+T)&=A\cos(\omega(t+T)+\phi)\\&=A\cos(\omega t+\omega T+\phi)\\&=A\cos(\omega t+2\pi+\phi)\\&=A\cos(\omega t+\phi)=x(t)\end{aligned}$$



Elementary Signals (3)

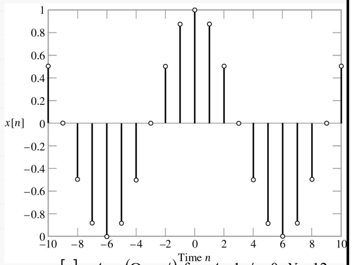
Discrete-time sinusoidal signal

$x[n] = A\cos(\Omega n + \phi)$,
$x[n+N] = A\cos(\Omega n + \Omega N + \phi)$,

For the above signal to be periodic,

$\Omega N = 2\pi m$ or $\Omega = \frac{2\pi m}{N}$ radians/cycle

for integer m, N



$x[n] = A\cos(\Omega n + \phi)$ for $A = 1, \phi = 0, N = 12$

Example 1.7 Discrete-time sinusoidal signals

$x_1[n] = \sin[5\pi n]$, $x_2[n] = \sqrt{3}\cos[5\pi n]$

(a) Find their common fundamental period.
(b) Express the composite sinusoidal signal $y[n] = x_1[n] + x_2[n]$ in the form $y[n] = A\cos(\Omega n + \phi)$.

Elementary Signals (4)

(a) The angular frequency of both $x_1[n]$ and $x_2[n]$

$\Omega = 5\pi$ rad/sec

Thus, $N = \frac{2\pi m}{\Omega} = \frac{2\pi m}{5\pi} = \frac{2m}{5}$

For $x_1[n]$ and $x_2[n]$ to be periodic, N must be an integer.
$m = 5, 10, 15, \cdots \rightarrow N = 2, 4, 6, \cdots$

(b) Recall the trigonometric identity

$A\cos(\Omega n + \phi) = A\cos(\Omega n)\cos(\phi) - A\sin(\Omega n)\sin(\phi)$

Letting $\Omega = 5\pi$, the right-hand side is of the same form as $x_1[n] + x_2[n]$

Therefore, $A\sin(\phi) = -1$ and $A\cos(\phi) = \sqrt{3}$

$\therefore \tan(\phi) = \frac{\sin(\phi)}{\cos(\phi)} = \frac{-1}{\sqrt{3}} \rightarrow \phi = -\frac{\pi}{6} \Longrightarrow A = \frac{-1}{\sin(-\pi/6)} = 2$

$\therefore y[n] = 2\cos\left(5\pi n - \frac{\pi}{6}\right)$

Elementary Signals (5)

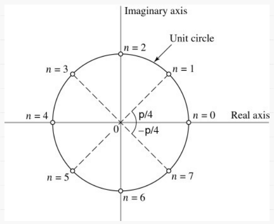
(Problem 1.16 ~ 1.18)

Relation bet. sinusoidal and complex exponential signals

$e^{j\theta} = \cos\theta + j\sin\theta$: Euler's identity; $B = Ae^{j\theta}$; $A\cos(\omega t + \theta) = \mathrm{Re}\{Be^{j\omega t}\}$
$x(t) = A\sin(\omega t + \theta)$; $A\sin(\omega t + \theta) = \mathrm{Im}\{Be^{j\omega t}\}$

$A\cos(\Omega n + \phi) = \mathrm{Re}\{Be^{j\Omega n}\}$
$A\sin(\Omega n + \phi) = \mathrm{Im}\{Be^{j\Omega n}\}$

For the case of $\Omega = \pi/4$ $\Longrightarrow$

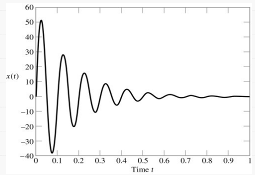


Elementary Signals (6)

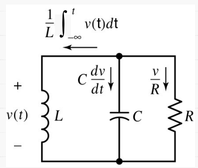
Exponentially damped sinusoidal signals

$x(t) = Ae^{-\alpha t}\sin(\omega t + \phi)$, $\alpha > 0$.

For $A = 60, \alpha = 6, \phi = 0$ $\Longrightarrow$



Example : Parallel RLC circuit



Integro-differential Eq.

$$C\frac{d}{dt}v(t) + \frac{1}{R}v(t) + \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau = 0$$

Solution

$$v(t) = V_0 e^{-t/(2CR)}\cos\omega_0 t, \quad t \ge 0; \ \omega_0 = \sqrt{\frac{1}{LC} - \frac{1}{4C^2R^2}}$$

• Discrete-time version of the exponentially damped sinusoidal signal

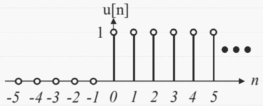
$x[n] = Br^n \sin[\Omega n + \phi]$, where $0 < |r| < 1$

(Problem 1.20 ~ 1.21)

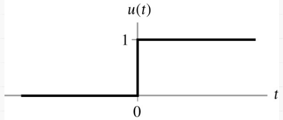
Elementary Signals (7)

Step Function

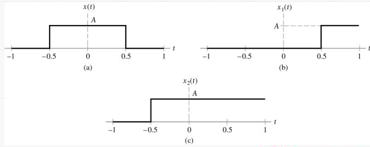
$$u[n]=\begin{cases}1, & n\ge 0\\ 0, & n<0\end{cases} \quad \Longleftrightarrow$$



$$u(t)=\begin{cases}1, & t\ge 0\\ 0, & t<0\end{cases} \quad \Longleftrightarrow$$



Example 1.8 Rectangular Pulse



Express $x(t)$ as a weighted sum of two step functions

$$x(t)=Au\left(t+\frac{1}{2}\right)-Au\left(t-\frac{1}{2}\right)$$

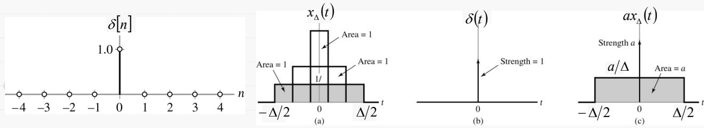
$$=x_2(t)-x_1(t)$$

Elementary Signals (8)

Impulse Function

$$\delta[n]=\begin{cases}1, & n=0\\ 0, & n\ne 0\end{cases}$$

$$\delta(t)=0 \quad \text{for } t\ne 0 \quad \text{and} \quad \int_{-\infty}^{\infty}\delta(t)\,dt=1$$



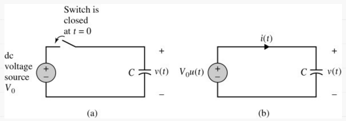
$$\delta(t)=\lim_{\Delta\to 0}x_\Delta(t)$$

- Impulse function vs. step function

$$\delta(t)=\frac{d}{dt}u(t) \quad \Longleftrightarrow \quad u(t)=\int_{-\infty}^{t}\delta(\tau)\,d\tau$$

Elementary Signals (9)

Example 1.10 RC Circuit



Determine the current that flows through the capacitor for $t\ge 0$.

$$v(t)=V_0u(t)$$

$$i(t)=C\frac{d}{dt}v(t)$$

$$i(t)=CV_0\frac{d}{dt}u(t)=CV_0\delta(t)$$

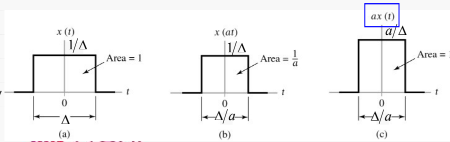
The other properties of Impulse function

$\boxed{\delta(t)=\delta(-t)}$: Even function

$\boxed{\int_{-\infty}^{\infty}x(t)\delta(t-t_0)\,dt=x(t_0)}$: Shifting property

$\boxed{\delta(at)=\frac{1}{a}\delta(t), \quad a>0}$

: Time-scaling property



Elementary Signals (10) : skip

Derivatives of the Impulse Function

- Impulse is the limiting form of a rectangular of duration Δ and amplitude $1/\Delta$
- First derivative
  - ✓ One impulse of strength $1/\Delta$, located at $t=-\Delta/2$
  - ✓ A second impulse of strength $-1/\Delta$, located at $t=\Delta/2$

$$\boxed{\delta^{(1)}(t)=\lim_{\Delta\to 0}\frac{1}{\Delta}\left(\delta(t+\Delta/2)-\delta(t-\Delta/2)\right)} \quad : \textit{doublet}$$

Properties of doublet

$$\int_{-\infty}^{\infty}\delta^{(1)}(t)\,dt=0; \quad \int_{-\infty}^{\infty}f(t)\delta^{(1)}(t-t_0)\,dt=\frac{d}{dt}f(t)\Big|_{t=t_0}$$

- Second derivative

$$\boxed{\frac{d^2}{dt^2}\delta(t)=\frac{d}{dt}\delta^{(1)}(t)=\lim_{\Delta\to 0}\frac{1}{\Delta}\left[\delta^{(1)}(t+\Delta/2)-\delta^{(1)}(t-\Delta/2)\right]}$$

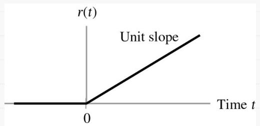
$$\int_{-\infty}^{\infty}f(t)\delta^{(2)}(t-t_0)\,dt=\frac{d^2}{dt^2}f(t)\Big|_{t=t_0} \quad ; \quad \int_{-\infty}^{\infty}f(t)\delta^{(n)}(t-t_0)\,dt=\frac{d^n}{dt^n}f(t)\Big|_{t=t_0}$$

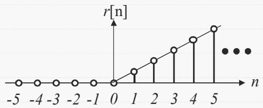
Elementary Signals (11)

Ramp Function

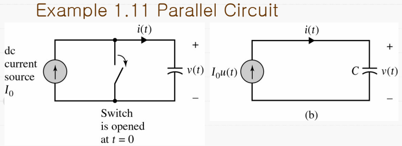
$$r(t)=\begin{cases} t, & t\ge 0 \\ 0, & t<0 \end{cases} \quad \Leftrightarrow \quad r(t)=tu(t)$$

$$r[n]=\begin{cases} n, & n\ge 0 \\ 0, & n<0 \end{cases} \quad \Leftrightarrow \quad r[n]=nu[n]$$





Example 1.11 Parallel Circuit



$$i(t)=I_0u(t)$$

$$v(t)=\frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau=\frac{1}{C}\int_{-\infty}^{t} I_0u(\tau)d\tau$$

$$=\begin{cases} 0, & t<0 \\ \frac{I_0}{C}tu(t), & t\ge 0 \end{cases} = \frac{I_0}{C}r(t)$$

Interconnected Systems

- Interconnected system → Interconnection of operations (mathematical term)

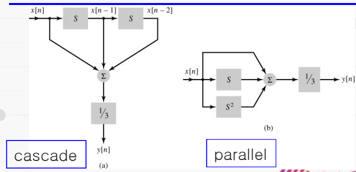
$$y(t)=\underbrace{H}_{\text{operator}}\{x(t)\} \qquad y[n]=H\{x[n]\}$$

$$x(t) \rightarrow \boxed{H} \rightarrow y(t) \qquad x[n] \rightarrow \boxed{H} \rightarrow y[n]$$

(a) (b)

Example 1.12 Moving-Average System

$$y[n]=\frac{1}{3}(x[n]+x[n-1]+x[n-2]) \longleftarrow \text{moving-average system}$$



Define *discrete-time-shift operator* : S^k

$$x[n] \rightarrow \boxed{S^k} \rightarrow x[n-k]$$

$$H=\frac{1}{3}(1+S+S^2)$$

Properties of Systems (1)

Stability

- Bounded-input, bounded-output (BIBO) stable if and only if every bounded input results in a bounded output
- For $y(t)=H\{x(t)\}$, operator H is BIBO stable if

$$|y(t)|\le M_y<\infty \quad \text{for all } t \quad \text{where } |x(t)|\le M_x<\infty \quad \text{for all } t$$

$$M_x,\ M_y : \text{some finite positive numbers}$$

Example 1.13 Moving-Average System

Show that the moving-average system is BIBO stable.

Assume that $|x[n]|\le M_x<\infty$ for all n.

$$|y[n]|=\frac{1}{3}|x[n]+x[n-1]+x[n-2]|\le\frac{1}{3}(|x[n]|+|x[n-1]|+|x[n-2]|)$$

$$\le\frac{1}{3}(M_x+M_x+M_x)=M_x$$

Properties of Systems (2)

Example 1.14 Unstable System

Show that the following system is unstable where $r>1$.

$$y[n]=r^n x[n]$$

Assume that $|x[n]|\le M_x<\infty$ for all n.

$$|y[n]|=|r^n x[n]|=|r^n|\cdot|x[n]|$$

With $r>1$, the multiplying factor r^n diverges for increasing n.

(Problem 1.26) (Refer to Fig. 1.52)

Memory

- A system is said to possess *memory* if its output depends on past or future values of the input
- A system is said to possess *memory-less* if its output depends only on the present value of input

Properties of Systems (3)

Examples :

$$i(t)=\frac{1}{R}v(t);\quad i(t)=\frac{1}{L}\int_{-\infty}^{t}v(\tau)d\tau$$

$$y[n]=\frac{1}{3}(x[n]+x[n-1]+x[n-2]);\quad y[n]=x^2[n]$$

(Problem 1.27 ~ 1.29)

Causality

- A system is said to be *causal* if the present value of output depends only on present or past values of input
- A system is said to be *non-causal* if its output depends on one or more future values of the input

$$y[n]=\frac{1}{3}(x[n]+x[n-1]+x[n-2]);\quad y[n]=\frac{1}{3}(\underline{x[n+1]}+x[n]+x[n-1])$$

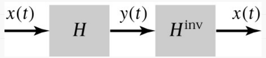
(Problem 1.30, 1.31)

Properties of Systems (4)

Invertibility

- A system is said to be *invertible* if the input of the system can be recovered from the output

$$H^{inv}\{y(t)\}=H^{inv}\{H\{x(t)\}\}=H^{inv}H\{x(t)\}$$



⟹ $H^{inv}H=I$: Identity operator

- H^{inv} : *inverse operator*, its associated system : *inverse system*
- In general, the problem of finding the inverse system is difficult one
- *Invertibility* is of importance in the design of communication systems.
- Distinct inputs should produce distinct outputs : one-to-one mapping!

Example 1.15 Inverse of System

$$y(t)=x(t-t_0)=S^{t_0}\{x(t)\}\qquad S^{t_0}:\text{time-shift operator}$$

$$S^{-t_0}\{y(t)\}=S^{-t_0}\{S^{t_0}\{x(t)\}\}=S^{-t_0}S^{t_0}\{x(t)\}$$

We require that $S^{-t_0}S^{t_0}=I$

Properties of Systems (5)

Example 1.16 Non-invertible System

$$y(t)=x^2(t)$$

To violate a necessary condition for invertibility : one-to-one mapping!
→ not invertible!!

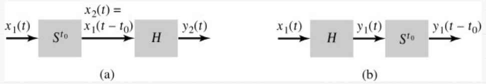
(Problem 1.32)

Time Invariance

- A system is said to be *time invariant* if a time delay or time advance of the input leads to an identical time shift in the output.
- Otherwise, called *time variant*

$$\boxed{y(t)=H\{x(t)\}\ \xleftrightarrow{TI}\ y(t-t_0)=H\{x(t-t_0)\}}$$

Properties of Systems (6)



$$y_2(t)=H\{x_1(t-t_0)\}=H\{S^{t_0}\{x_1(t)\}\}=HS^{t_0}\{x_1(t)\}$$

$$y_1(t-t_0)=S^{t_0}\{y_1(t)\}=S^{t_0}\{H\{x_1(t)\}\}=S^{t_0}H\{x_1(t)\}$$

- For time invariant system, $\boxed{HS^{t_0}=S^{t_0}H}$
- Two operators must *commute* with each other for all t_0

Example 1.17 Inductor

$$y_1(t)=\frac{1}{L}\int_{-\infty}^{t}x_1(\tau)d\tau \Longleftarrow$$

- Input : voltage across an inductor
- Output : current through the inductor

$$y_2(t)=\frac{1}{L}\int_{-\infty}^{t}x_1(\tau-t_0)d\tau \iff y_1(t-t_0)=\frac{1}{L}\int_{-\infty}^{t-t_0}x_1(\tau)d\tau$$

Put $\tau'=\tau-t_0$, two equations become equal to each other

Properties of Systems (7)

Example 1.18 Thermistor

$$y_1(t) = \frac{x_1(t)}{R(t)}$$

- Resistance varies with temperature
- Input : voltage across a thermistor
- Output : current through the thermistor

$$y_2(t) = \frac{x_1(t-t_0)}{R(t)} \Longleftrightarrow y_1(t-t_0) = \frac{x_1(t-t_0)}{R(t-t_0)}$$

Since, in general, $R(t) \neq R(t-t_0)$ for $t_0 \neq 0$

$y_1(t-t_0) \neq y_2(t)$ for $t_0 \neq 0$ ➡ Time variant!!

(Problem 1.33)

Linearity

- A system is said to be *linear* if it satisfies the following two properties
  - ✓ Principle of superposition
    $y_1(t) = H\{x_1(t)\}, y_2(t) = H\{x_2(t)\}$ ➡ $y_1(t) + y_2(t) = H\{x_1(t) + x_2(t)\}$
  - ✓ Homogeneity : Whenever input is scaled by a, output is scaled by exactly the same constant factor a.

Properties of Systems (8)

$$y(t) = H\left\{\sum_{i=1}^{N} a_i x_i(t)\right\} = \sum_{i=1}^{N} a_i H\{x_i(t)\} = \sum_{i=1}^{N} a_i y_i(t)$$

Linearity

(Problem 1.34 ~ 1.36)

Example 1.19 Linear Discrete-Time System

$y[n] = nx[n]$ Show that this system is linear.

$$x[n] = \sum_{i=1}^{N} a_i x_i[n] \Longleftrightarrow y[n] = n\sum_{i=1}^{N} a_i x_i[n] = \sum_{i=1}^{N} a_i n x_i[n] = \sum_{i=1}^{N} a_i y_i[n]$$

∴ it is a linear system

Example 1.20 Non-linear Continuous-Time System $y(t) = x(t)x(t-1)$

$$x(t) = \sum_{i=1}^{N} a_i x_i(t) \Longleftrightarrow y(t) = \sum_{i=1}^{N} a_i x_i(t) \sum_{j=1}^{N} a_j x_j(t-1) = \sum_{i=1}^{N}\sum_{j=1}^{N} a_i a_j x_i(t) x_j(t-1)$$

$$\neq \sum_{i=1}^{N} a_i y_i(t)$$ ∴ it is non-linear!

Noise

- Noise : unwanted signal
  - ✓ Tend to disturb the operation of a system
  - ✓ We have incomplete control over it
- External sources of noise : atmospheric noise, galactic noise, and human-made noise
- Internal sources of noise : electrical noise, etc.

Thermal Noise

- To arise from the random motion of electrons in a conductor
- Time-averaged value

$$\bar{v} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} v(t)\,dt$$

- Time-average-squared value

$$\overline{v^2} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} v^2(t)\,dt \Longleftrightarrow \overline{v^2} = 4kT_{abs}R\Delta f \text{ volt}^2, \quad \overline{i^2} = 4kT_{abs}G\Delta f \text{ amps}^2$$

Homework #1

- To solve more than 10 additional problems but the following are mandatory
  - 42, 44, 46, 47, 52, 53, 54
- Due date :