Chapter 22

1. a. left: $d_1 = \frac{3}{16}'' = 0.1875''$, $d_2 = 1''$

$$\text{Value} = 10^3 \times 10^{0.1875''/1''}$$
$$= 10^3 \times 1.54$$
$$= \mathbf{1.54\ kHz}$$

center: $d_1 = \frac{1}{2}'' = 0.5''$, $d_2 = 1''$

$$\text{Value} = 10^3 \times 10^{0.5/1}$$
$$= 10^3 \times 10^{.5}$$
$$= 10^3 \times 3.16$$
$$= \mathbf{3.16\ kHz}$$

right: $d_1 = \frac{3}{4}'' = 0.75''$, $d_2 = 1''$

$$\text{Value} = 10^3 \times 10^{0.75''/1''}$$
$$= 10^3 \times 5.623$$
$$= \mathbf{5.62\ kHz}$$

b. bottom: $d_1 = \frac{5}{16}'' = 0.3125''$, $d_2 = \frac{15}{16}'' = 0.9375''$

$$\text{Value} = 10^{-1} \times 10^{0.3125''/0.9375''} = 10^{-1} \times 10^{0.333}$$
$$= 10^{-1} \times 2.153$$
$$= \mathbf{0.22\ V}$$

center: $d_1 = \frac{7.5}{16}'' = 0.469''$, $d_2 = 0.9375''$

$$\text{Value} = 10^{-1} \times 10^{0.469/0.9375}$$
$$= 10^{-1} \times 10^{0.5}$$
$$= 10^{-1} \times 3.16$$
$$= \mathbf{0.316\ V}$$

top: $d_1 = \frac{11}{16}'' = 0.6875''$, $d_2 = 0.9375''$

$$\text{Value} = 10^{-1} \times 10^{0.6875''/0.9375''} = 10^{-1} \times 10^{0.720}$$
$$= 10^{-1} \times 5.248$$
$$= \mathbf{0.52\ V}$$

2. a. **5** b. **−4** c. **8** d. **−6**

e. **1.30** f. **3.94** g. **4.75** h. **−0.498**

3. a. **1000** b. **10^{12}** c. **1.59** d. **1.1**

e. **10^{10}** f. **1513.56** g. **10.02** h. **1,258,925.41**

4. a. **11.51** b. **−9.21** c. **2.996** d. **9.07**

5. $\log_{10} 48 =$ **1.68**

$\log_{10} 8 + \log_{10} 6 = 0.903 + 0.778 = \mathbf{1.68}$

6. $\log_{10} 0.2 = \mathbf{-0.699}$
$\log_{10} 18 - \log_{10} 90 = 1.255 - 1.954 = \mathbf{-0.699}$

7. $\log_{10} 0.5 = \mathbf{-0.30}$
$-\log_{10} 2 = -(0.301) = \mathbf{-0.30}$

8. $\log_{10} 27 = \mathbf{1.43}$
$3 \log_{10} 3 = 3(0.4771) = \mathbf{1.43}$

9. a. $\text{bels} = \log_{10} \frac{P_2}{P_1} = \log_{10} \frac{280\text{ mW}}{4\text{ mW}} = \log_{10} 70 = \mathbf{1.85}$

b. $\text{dB} = 10 \log_{10} \frac{P_2}{P_1} = 10(\log_{10} 70) = 10(1.845) = \mathbf{18.45}$

10. $\text{dB} = 10 \log_{10} \frac{P_2}{P_1}$

$6\text{ dB} = 10 \log_{10} \frac{100\text{ W}}{P_1}$

$0.6 = \log_{10} x$

$x = 3.981 = \frac{100\text{ W}}{P_1}$

$P_1 = \frac{100\text{ W}}{3.981} = \mathbf{25.12\ W}$

11. $\text{dB} = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{40\text{ W}}{2\text{ W}} = 10 \log_{10} 20 = \mathbf{13.01}$

12. $\text{dB}_m = 10 \log_{10} \frac{P}{1\text{ mW}}$

$\text{dB}_m = 10 \log_{10} \frac{120\text{ mW}}{1\text{ mW}} = 10 \log_{10} 120 = \mathbf{20.79}$

13. $\text{dB}_v = 20 \log_{10} \frac{V_2}{V_1} = 20 \log_{10} \frac{8.4\text{ V}}{0.1\text{ V}} = 20 \log_{10} 84 = \mathbf{38.49}$

14. $\text{dB}_v = 20 \log_{10} \frac{V_2}{V_1}$

$22 = 20 \log_{10} \frac{V_o}{20\text{ mV}}$

$1.1 = \log_{10} x$

$x = 12.589 = \frac{V_o}{20\text{ mV}}$

$V_o = \mathbf{251.79\ mV}$

15. $\text{dB}_s = 20 \log_{10} \dfrac{P}{0.0002\,\mu\text{bar}}$

$\text{dB}_s = 20 \log_{10} \dfrac{0.001\,\mu\text{bar}}{0.0002\,\mu\text{bar}} = \mathbf{13.98}$

$\text{dB}_s = 20 \log_{10} \dfrac{0.016\,\mu\text{bar}}{0.0002\,\mu\text{bar}} = \mathbf{38.06}$

Increase = $\mathbf{24.08\ dB_s}$

16. $60\ \text{dB}_s \Rightarrow 90\ \text{dB}_s$
quiet loud

$60\ \text{dB}_s = 20 \log_{10} \dfrac{P_1}{0.002\,\mu\text{bar}} = 20 \log_{10} x$

$3 = \log_{10} x$

$x = \mathbf{1000}$

$90\ \text{dB}_s = 20 \log_{10} \dfrac{P_2}{0.002\,\mu\text{bar}} = 20 \log_{10} y$

$4.5 = \log_{10} y$

$y = \mathbf{31.623 \times 10^3}$

$$\frac{x}{y} = \frac{\dfrac{P_1}{0.002\,\cancel{\mu\text{bar}}}}{\dfrac{P_2}{0.002\,\cancel{\mu\text{bar}}}} = \frac{P_1}{P_2} = \frac{\cancel{10^3}}{31.623 \times \cancel{10^3}}$$

and $P_2 = \mathbf{31.62\ P_1}$

17. –

18. a. $8\ \text{dB} = 20 \log_{10} \dfrac{V_2}{0.775\ \text{V}}$

$0.4 = \log_{10} \dfrac{V_2}{0.775\ \text{V}}$

$\dfrac{V_2}{0.775\ \text{V}} = 2.512$

$V_2 = (2.512)(0.775\ \text{V}) = 1.947\ \text{V}$

$P = \dfrac{V^2}{R} = \dfrac{(1.947\ \text{V})^2}{600\ \Omega} = \mathbf{6.32\ mW}$

b. $-5\ \text{dB} = 20 \log_{10} \dfrac{V_2}{0.775\ \text{V}}$

$-0.25 = \log_{10} \dfrac{V_2}{0.775\ \text{V}}$

$\dfrac{V_2}{0.775\ \text{V}} = 0.562$

$V_2 = (0.562)(0.775\ \text{V}) = 0.436\ \text{V}$

$P = \dfrac{V^2}{R} = \dfrac{(0.436\ \text{V})^2}{600\ \Omega} = \mathbf{0.32\ mW}$

19. a. $A_\upsilon = \dfrac{\mathbf{V}_o}{\mathbf{V}_i} = \dfrac{X_C}{\sqrt{R^2 + X_C^2}} \angle -90° + \tan^{-1} X_C/R = \dfrac{1}{\sqrt{\left(\dfrac{R}{X_C}\right)^2 + 1}} \angle -\tan^{-1} R/X_C$

$f_c = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi(2.2\text{ k}\Omega)(0.02\,\mu\text{F})} = \mathbf{3617.16\ Hz}$

$f = f_c$: $A_\upsilon = \dfrac{V_o}{V_i} = \mathbf{0.707}$

$f = 0.1f_c$: At f_c, $X_C = R = 2.2\text{ k}\Omega$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi 0.1 f_c C} = \dfrac{1}{0.1}\left[\dfrac{1}{2\pi f_c C}\right] = 10[2.2\text{ k}\Omega] = 22\text{ k}\Omega$

$A_\upsilon = \dfrac{1}{\sqrt{\left(\dfrac{R}{X_C}\right)^2 + 1}} = \dfrac{1}{\sqrt{\left(\dfrac{2.2\text{ k}\Omega}{22\text{ k}\Omega}\right)^2 + 1}} = \dfrac{1}{\sqrt{(.1)^2 + 1}} = \mathbf{0.995}$

$f = 0.5f_c = \dfrac{1}{2} f_c$: $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi\left(\dfrac{f_c}{2}\right)C} = 2\left[\dfrac{1}{2\pi f_c C}\right] = 2[2.2\text{ k}\Omega] = 4.4\text{ k}\Omega$

$A_\upsilon = \dfrac{1}{\sqrt{\left(\dfrac{2.2\text{ k}\Omega}{4.4\text{ k}\Omega}\right)^2 + 1}} = \dfrac{1}{\sqrt{(0.5)^2 + 1}} = \mathbf{0.894}$

$f = 2f_c$: $X_C = \dfrac{1}{2\pi(2f_c)C} = \dfrac{1}{2}\left[\dfrac{1}{2\pi f_c C}\right] = \dfrac{1}{2}[2.2\text{ k}\Omega] = 1.1\text{ k}\Omega$

$A_\upsilon = \dfrac{1}{\sqrt{\left(\dfrac{2.2\text{ k}\Omega}{1.1\text{ k}\Omega}\right)^2 + 1}} = \dfrac{1}{\sqrt{(2)^2 + 1}} = \mathbf{0.447}$

$f = 10f_c$: $X_C = \dfrac{1}{2\pi(10f_c)C} = \dfrac{1}{10}\left[\dfrac{1}{2\pi f_c C}\right] = \dfrac{1}{10}[2.2\text{ k}\Omega] = 0.22\text{ k}\Omega$

$A_\upsilon = \dfrac{1}{\sqrt{\left(\dfrac{2.2\text{ k}\Omega}{0.22\text{ k}\Omega}\right)^2 + 1}} = \dfrac{1}{\sqrt{(10)^2 + 1}} = \mathbf{0.0995}$

b. $\theta = -\tan^{-1} R/X_C$

$f = f_c$: $\theta = -\tan^{-1} = \mathbf{-45°}$

$f = 0.1f_c$: $\theta = -\tan^{-1} 2.2\text{ k}\Omega/22\text{ k}\Omega = -\tan^{-1}\dfrac{1}{10} = \mathbf{-5.71°}$

$f = 0.5f_c$: $\theta = -\tan^{-1} 2.2\text{ k}\Omega/4.4\text{ k}\Omega = -\tan^{-1}\dfrac{1}{2} = \mathbf{-26.57°}$

$f = 2f_c$: $\theta = -\tan^{-1} 2.2\text{ k}\Omega/1.1\text{ k}\Omega = -\tan^{-1} 2 = \mathbf{-63.43°}$

$f = 10f_c$: $\theta = -\tan^{-1} 2.2\text{ k}\Omega/0.22\text{ k}\Omega = -\tan^{-1} 10 = \mathbf{-84.29°}$

20. a. $f_c = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi(2.2\text{ k}\Omega)(0.1\ \mu\text{F})} = 723.43\text{ Hz}$

$f = 2f_c = 1.45\text{ kHz}$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(1.45\text{ kHz})(0.1\ \mu\text{F})} = 1.1\text{ k}\Omega$

$A_v = \dfrac{V_o}{V_i} = \dfrac{X_C}{\sqrt{R^2 + X_C^2}} = \dfrac{1.1\text{ k}\Omega}{\sqrt{(2.2\text{ k}\Omega)^2 + (1\text{ k}\Omega)^2}} = 0.4472$

$V_o = 0.4472V_i = 0.4472(10\text{ mV}) = \mathbf{4.47\ mV}$

b. $f = \dfrac{1}{10}f_c = \dfrac{1}{10}(723.43\text{ Hz}) = 72.34\text{ Hz}$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(72.34\text{ Hz})(0.1\ \mu\text{F})} = 22\text{ k}\Omega$

$A_v = \dfrac{V_o}{V_i} = \dfrac{X_C}{\sqrt{R^2 + X_C^2}} = \dfrac{22\text{ k}\Omega}{\sqrt{(2.2\text{ k}\Omega)^2 + (22\text{ k}\Omega)^2}} = \dfrac{22\text{ k}\Omega}{22.11\text{ k}\Omega} = 0.995$

$V_o = 0.995V_i = 0.995(10\text{ mV}) = \mathbf{9.95\ mV}$

c. Yes, at $f = f_c$, $V_o = 7.07$ mV

at $f = \dfrac{1}{10}f_c$, $V_o = 9.95$ mV (much higher)

at $f = 2f_c$, $V_o = 4.47$ mV (much lower)

21. $f_c = 500\text{ Hz} = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi(1.2\text{ k}\Omega)C}$

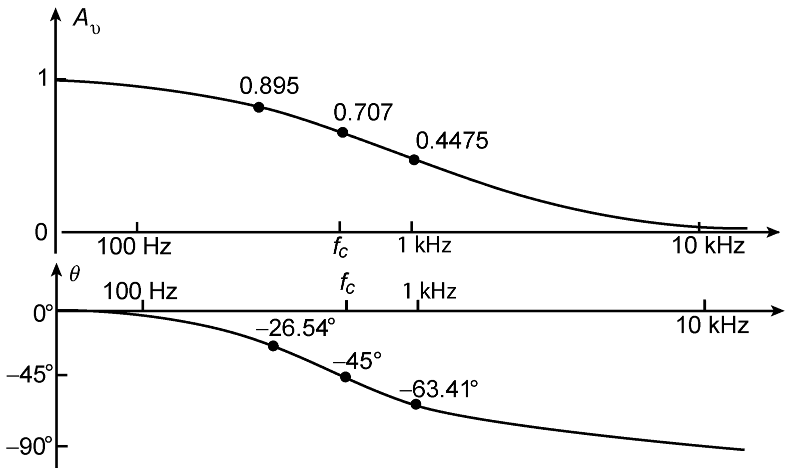
$C = \dfrac{1}{2\pi Rf_c} = \dfrac{1}{2\pi(1.2\text{ k}\Omega)(500\text{ Hz})} = \mathbf{0.265\ \mu F}$

$A_v = \dfrac{V_o}{V_i} = \dfrac{1}{\sqrt{\left(\dfrac{R}{X_C}\right)^2 + 1}}$

At $f = 250$ Hz, $X_C = 2402.33\ \Omega$ and $A_v = 0.895$
At $f = 1000$ Hz, $X_C = 600.58\ \Omega$ and $A_v = 0.4475$
$\theta = -\tan^{-1}R/X_C$

At $f = 250\text{ Hz} = \dfrac{1}{2}f_c$, $\theta = -26.54°$

At $f = 1\text{ kHz} = 2f_c$, $\theta = -63.41°$



22. a. $$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(4.7\text{ k}\Omega)(500\text{ pF})} = \mathbf{67.73\ kHz}$$

b. $f = 0.1\, f_c = 0.1(67.726\text{ kHz}) \cong 6.773\text{ kHz}$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(6.773\text{ kHz})(500\text{ pF})} = 46.997\text{ k}\Omega$$

$$A_\upsilon = \frac{V_o}{V_i} = \frac{X_C}{\sqrt{R^2 + X_C^2}} = \frac{46.997\text{ k}\Omega}{\sqrt{(4.7\text{ k}\Omega)^2 + (46.997\text{ k}\Omega)^2}} = \mathbf{0.995} \cong 1$$

c. $f = 10f_c = 677.26\text{ kHz}$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(677.26\text{ kHz})(500\text{ pF})} \cong 470\ \Omega$$

$$A_\upsilon = \frac{V_o}{V_i} = \frac{X_C}{\sqrt{R^2 + X_C^2}} = \frac{470\ \Omega}{\sqrt{(4.7\text{ k}\Omega)^2 + (470\ \Omega)^2}} = \mathbf{0.0995} \cong 0.1$$

d. $$A_\upsilon = \frac{V_o}{V_i} = 0.01 = \frac{X_C}{\sqrt{R^2 + X_C^2}}$$

$$\sqrt{R^2 + X_C^2} = \frac{X_C}{0.01} = 100\ X_C$$

$$R^2 + X_C^2 = 10^4\ X_C^2$$

$$R^2 = 10^4\ X_C^2 - X_C^2 = 9{,}999\ X_C^2$$

$$X_C = \frac{R}{\sqrt{9{,}999}} = \frac{4.7\text{ k}\Omega}{99.995} \cong 47\ \Omega$$

$$X_C = \frac{1}{2\pi f C} \Rightarrow f = \frac{1}{2\pi X_C C} = \frac{1}{2\pi(47\ \Omega)(500\text{ pF})} = \mathbf{6.77\ MHz}$$

23. a. $A_v = \dfrac{\mathbf{V}_o}{\mathbf{V}_i} = \dfrac{R}{\sqrt{R^2 + X_C^2}} \angle \tan^{-1} X_C/R = \dfrac{1}{\sqrt{1 + \left(\dfrac{X_C}{R}\right)^2}} \angle \tan^{-1} X_C/R$

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(2.2\text{ k}\Omega)(0.02\,\mu\text{F})} = \mathbf{3.62\text{ kHz}}$$

$f = f_c$: $A_v = \dfrac{V_o}{V_i} = \mathbf{0.707}$

$f = 2f_c$: At f_c, $X_C = R = 2.2\text{ k}\Omega$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(2f_c)C} = \frac{1}{2}\left[\frac{1}{2\pi(2f_c)C}\right] = \frac{1}{2}[2.2\text{ k}\Omega] = 1.1\text{ k}\Omega$$

$$A_v = \frac{1}{\sqrt{1 + \left(\dfrac{1.1\text{ k}\Omega}{2.2\text{ k}\Omega}\right)^2}} = \mathbf{0.894}$$

$f = \dfrac{1}{2}f_c$: $X_C = \dfrac{1}{2\pi\left(\dfrac{f_c}{2}\right)C} = 2\left[\dfrac{1}{2\pi f_c C}\right] = 2[2.2\text{ k}\Omega] = 4.4\text{ k}\Omega$

$$A_v = \frac{1}{\sqrt{1 + \left(\dfrac{4.4\text{ k}\Omega}{2.2\text{ k}\Omega}\right)^2}} = \mathbf{0.447}$$

$f = 10f_c$: $X_C = \dfrac{1}{2\pi(10f_c)C} = \dfrac{1}{10}\left[\dfrac{1}{2\pi f_c C}\right] = \dfrac{2.2\text{ k}\Omega}{10} = 0.22\text{ k}\Omega$

$$A_v = \frac{1}{\sqrt{1 + \left(\dfrac{0.22\text{ k}\Omega}{2.2\text{ k}\Omega}\right)^2}} = \mathbf{0.995}$$

$f = \dfrac{1}{10}f_c$: $X_C = \dfrac{1}{2\pi\left(\dfrac{f_c}{10}\right)C} = 10\left[\dfrac{1}{2\pi f_c C}\right] = 10[2.2\text{ k}\Omega] = 22\text{ k}\Omega$

$$A_v = \frac{1}{\sqrt{1 + \left(\dfrac{22\text{ k}\Omega}{2.2\text{ k}\Omega}\right)^2}} = \mathbf{0.0995}$$

b. $f = f_c$, $\theta = \mathbf{45^\circ}$

$f = 2f_c$, $\theta = \tan^{-1}(X_C/R) = \tan^{-1} 1.1\text{ k}\Omega/2.2\text{ k}\Omega = \tan^{-1}\dfrac{1}{2} = \mathbf{26.57^\circ}$

$f = \dfrac{1}{2}f_c$, $\theta = \tan^{-1}\dfrac{4.4\text{ k}\Omega}{2.2\text{ k}\Omega} = \tan^{-1} 2 = \mathbf{63.43^\circ}$

$f = 10f_c$, $\theta = \tan^{-1}\dfrac{0.22\text{ k}\Omega}{2.2\text{ k}\Omega} = \mathbf{5.71^\circ}$

$$f = \frac{1}{10} f_c, \qquad \theta = \tan^{-1} \frac{22\text{ k}\Omega}{2.2\text{ k}\Omega} = \mathbf{84.29°}$$

24. a. $f = f_c$: $A_\upsilon = \frac{V_o}{V_i} = \mathbf{0.707}$

b. $$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(120\text{ k}\Omega)(47\text{ pF})} = 28.22\text{ kHz}$$

$f = 4f_c = 4(28.22\text{ kHz}) = 112.88\text{ kHz}$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(112.88\text{ kHz})(47\text{ pF})} = 30\text{ k}\Omega$$

$$A_\upsilon = \frac{V_o}{V_i} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{120\text{ k}\Omega}{\sqrt{(120\text{ k}\Omega)^2 + (30\text{ k}\Omega)^2}} = \mathbf{0.970}\text{ (significant rise)}$$

c. $f = 100f_c = 100(28.22\text{ kHz}) = 2.82\text{ MHz}$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(2.82\text{ MHz})(47\text{ pF})} = 1.2\text{ k}\Omega$$

$$A_\upsilon = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{120\text{ k}\Omega}{\sqrt{(120\text{ k}\Omega)^2 + (1.2\text{ k}\Omega)^2}} = 1$$

d. At $f = f_c$, $V_o = 0.707V_i = 0.707(10\text{ mV}) = 7.07\text{ mV}$

$$P_o = \frac{V_o^2}{R} = \frac{(7.07\text{ mV})^2}{120\text{ k}\Omega} \cong \mathbf{0.417\text{ nW}}$$

25. $$\mathbf{A}_\upsilon = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{\sqrt{1 + \left(\frac{X_C}{R}\right)^2}} \angle \tan^{-1} X_C/R$$

$$f_c = \frac{1}{2\pi RC} \Rightarrow R = \frac{1}{2\pi f_c C} = \frac{1}{2\pi(2\text{ kHz})(0.1\,\mu\text{F})} = 795.77\ \Omega$$

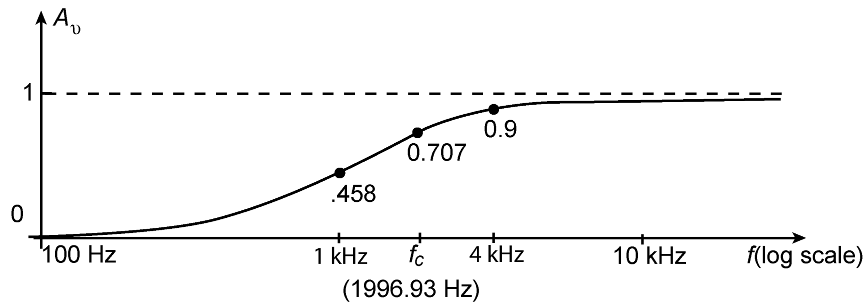
$$R = 795.77\ \Omega \Rightarrow \underbrace{750\ \Omega + 47\ \Omega}_{\text{nominal values}} = 797\ \Omega$$

$$\therefore f_c = \frac{1}{2\pi(797\ \Omega)(0.1\,\mu\text{F})} = \mathbf{1996.93\text{ Hz}}\text{ using nominal values}$$

At $f = 1\text{ kHz}$, $A_\upsilon = 0.458$
$f = 4\text{ kHz}$, $A_\upsilon \cong 0.9$

$$\theta = \tan^{-1} \frac{X_C}{R}$$

$f = 1\text{ kHz}$, $\theta = 63.4°$
$f = 4\text{ kHz}$, $\theta = 26.53°$



26. a. $f_c = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi(100\text{ k}\Omega)(20\text{ pF})} = \mathbf{79.58\text{ kHz}}$

b. $f = 0.01f_c = 0.01(79.577\text{ kHz}) = 0.7958\text{ kHz} \cong 796\text{ Hz}$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(796\text{ Hz})(20\text{ pF})} = 9.997\text{ M}\Omega$$

$$A_v = \frac{V_o}{V_i} = \frac{R}{\sqrt{R^2+X_C^2}} = \frac{100\text{ k}\Omega}{\sqrt{(100\text{ k}\Omega)^2+(9.997\text{ M}\Omega)^2}} = \mathbf{0.01} \cong 0$$

c. $f = 100f_c = 100(79.577\text{ kHz}) \cong 7.96\text{ MHz}$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(7.96\text{ MHz})(20\text{ pF})} = 999.72\ \Omega$$

$$A_v = \frac{V_o}{V_i} = \frac{R}{\sqrt{R^2+X_C^2}} = \frac{100\text{ k}\Omega}{\sqrt{(100\text{ k}\Omega)^2+(999.72\ \Omega)^2}} = \mathbf{0.99995} \cong 1$$

d. $A_v = \dfrac{V_o}{V_i} = 0.5 = \dfrac{R}{\sqrt{R^2+X_C^2}}$

$$\sqrt{R^2+X_C^2} = 2R$$

$$R^2 + X_C^2 = 4R^2$$

$$X_C^2 = 4R^2 - R^2 = 3R^2$$

$$X_C = \sqrt{3R^2} = \sqrt{3}R = \sqrt{3}\,(100\text{ k}\Omega) = 173.2\text{ k}\Omega$$

$$X_C = \frac{1}{2\pi fC} \Rightarrow f = \frac{1}{2\pi X_C C} = \frac{1}{2\pi(173.2\text{ k}\Omega)(20\text{ pF})}$$

$f = \mathbf{45.95\text{ kHz}}$

27. a. low-pass section: $f_{c_1} = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi(0.1\text{ k}\Omega)(2\ \mu\text{F})} = \mathbf{795.77\text{ Hz}}$

high-pass section: $f_{c_2} = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi(10\text{ k}\Omega)(8200\text{ pF})} = \mathbf{1.94\text{ Hz}}$

For the analysis to follow, it is assumed $(R_2 + jX_{C_2}) \parallel R_1 \cong R_1$ for all frequencies of interest.

At f_{c_1} = 795.77 Hz:

$V_{R_1} = 0.707\ V_i$

$$X_{C_2} = \frac{1}{2\pi f C_2} = 24.39\ \text{k}\Omega$$

$$|V_o| = \frac{24.39\ \text{k}\Omega(V_{R_1})}{\sqrt{(10\ \text{k}\Omega)^2 + (24.39\ \text{k}\Omega)^2}} = 0.925\ V_{R_1}$$

$V_o = (0.925)(0.707\ V_i) = \mathbf{0.654}\ \boldsymbol{V_i}$

At f_{c_2} = 1.94 kHz:

$V_o = 0.707\ V_{R_1}$

$$X_{C_1} = \frac{1}{2\pi f C_1} = 41\ \Omega$$

$$|V_{R_1}| = \frac{R_1 V_i}{\sqrt{R_1^2 + X_{C_1}^2}} = \frac{100\ \Omega(V_i)}{\sqrt{(100\ \Omega)^2 + (41\ \Omega)^2}} = 0.925\ V_i$$

$|V_o| = (0.707)(0.925\ V_i) = \mathbf{0.64}\ \boldsymbol{V_i}$

$$\text{At } f = 795.77\ \text{Hz} + \frac{(1.94\ \text{kHz} - 795.77\ \text{Hz})}{2} = 1.37\ \text{kHz}$$

$X_{C_1} = 58.1\ \Omega$, $X_{C_2} = 14.17\ \text{k}\Omega$

$$|V_{R_1}| = \frac{100\ \Omega(V_i)}{\sqrt{(100\ \Omega)^2 + (58.1\ \Omega)^2}} = 0.864\ V_i$$

$$|V_o| = \frac{14.17\ \text{k}\Omega\left(V_{R_1}\right)}{\sqrt{(10\ \text{k}\Omega)^2 + (14.17\ \text{k}\Omega)^2}} = 0.817\ V_{R_1}$$

$V_o = 0.817(0.864\ V_i) = 0.706 V_i$

and $A_v = \dfrac{V_o}{V_i}$ = **0.706** (≅ maximum value)

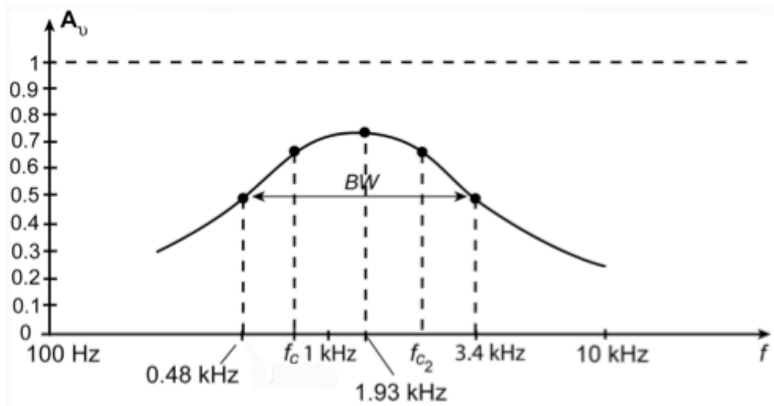
After plotting the points it was determined that the gain should also be determined at f = 500 Hz and 4 kHz:

f = 500 Hz: $X_{C_1} = 159.15\ \Omega$, $X_{C_2} = 38.82\ \text{k}\Omega$,
$V_{R_1} = 0.532\ V_i$, $V_o = 0.968\ V_{R_1}$
$V_o = \mathbf{0.515}\ \boldsymbol{V_i}$

f = 4 kHz: $X_{C_1} = 19.89\ \Omega$, $X_{C_2} = 4.85\ \text{k}\Omega$,
$V_{R_1} = 0.981\ V_i$, $V_o = 0.437\ V_{R_1}$
$V_o = \mathbf{0.429}\ \boldsymbol{V_i}$

b. Using 0.707(.706) ≅ 0.5 to define the bandwidth
BW ≅ 3.4 kHz – 0.48 kHz = 2.92 kHz
and BW ≅ **2.9 kHz**

with $f_{center} = 480\text{ Hz} + \left(\dfrac{2.9\text{ kHz}}{2}\right) = \mathbf{1930\ Hz}$



28. $f_1 = \dfrac{1}{2\pi R_1 C_1} = 4\text{ kHz}$

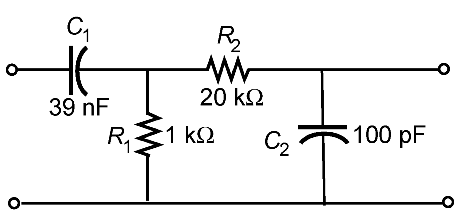
Choose $R_1 = 1\text{ k}\Omega$

$$C_1 = \frac{1}{2\pi f_1 R_1} = \frac{1}{2\pi(4\text{ kHz})(1\text{ k}\Omega)} = 39.8\text{ nF} \therefore \text{ Use } \mathbf{39\ nF}$$

$f_2 = \dfrac{1}{2\pi R_2 C_2} = 80\text{ kHz}$

Choose $R_2 = 20\text{ k}\Omega$

$$C_2 = \frac{1}{2\pi f_2 R_2} = \frac{1}{2\pi(80\text{ kHz})(20\text{ k}\Omega)} = 99.47\text{ pF} \therefore \text{ Use 100 pF}$$



Center frequency $= 4\text{ kHz} + \dfrac{80\text{ kHz} - 4\text{ kHz}}{2} = 42\text{ kHz}$

At $f = 42$ kHz, $X_{C_1} = 97.16\ \Omega$, $X_{C_2} = 37.89\text{ k}\Omega$

Assuming $\boldsymbol{Z}_2 \gg \boldsymbol{Z}_1$

$|V_{R_1}| = \dfrac{R_1(V_i)}{\sqrt{R_1^2 + X_{C_1}^2}} = 0.995V_i$

$|V_o| = \dfrac{X_{C_2}(V_{R_1})}{\sqrt{R_2^2 + X_{C_1}^2}} = 0.884V_i$

$V_o = 0.884V_{R_1} = 0.884(0.995V_i) = \mathbf{0.88\ V_i}$

as $f = f_1$: $V_{R_1} = 0.707V_i$, $X_{C_2} = 221.05\text{ k}\Omega$

and $V_o = 0.996 V_{R_1}$

so that $V_o = 0.996 V_{R_1} = 0.996(0.707 V_i) = 0.704 V_i$

Although $A_v = 0.88$ is less than the desired level of 1, f_1 and f_2 do define a band of frequencies for which $A_v \geq 0.7$ and the power to the load is significant.

29. a. $f_s = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(4.7\text{ mH})(560\text{ pF})}} = \mathbf{98.1\ kHz}$

b. $Q_s = \dfrac{X_L}{R + R_\ell} = \dfrac{2\pi(98.1\text{ kHz})(4.7\text{ mH})}{160\ \Omega + 12\ \Omega} = \mathbf{16.84}$

$BW = \dfrac{f_s}{Q_s} = \dfrac{98.1\text{ kHz}}{16.84} = \mathbf{5.83\ kHz}$

c. At $f = f_s$: $V_{o_{max}} = \dfrac{R}{R + R_\ell} V_i = \dfrac{160\ \Omega(1\text{ V})}{172\ \Omega} = 0.93\text{ V}$ and $A_v = \dfrac{V_o}{V_i} = \mathbf{0.93}$

Since $Q_s \geq 10$, $f_1 = f_s - \dfrac{BW}{2} = 98.1\text{ kHz} - \dfrac{5.83\text{ kHz}}{2} = 95.19\text{ kHz}$

$f_2 = f_s + \dfrac{BW}{2} = 101.02\text{ kHz}$

At $f = 95.19$ kHz: $X_L = 2\pi fL = 2\pi(95.19\text{ kHz})(4.7\text{ mH}) = 2.81\text{ k}\Omega$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(95.19\text{ kHz})(560\text{ pF})} = 2.99\text{ k}\Omega$

$\mathbf{V}_o = \dfrac{160\ \Omega(1\text{ V} \angle 0°)}{172 + j2.81\text{ k}\Omega - j2.99\text{ k}\Omega} = \dfrac{160\text{ V} \angle 0°}{172 - j180}$

$= \dfrac{160\text{ V} \angle 0°}{248.97 \angle -46.30°} = \mathbf{0.643\ V\angle 46.30°}$

At $f = 101.02$ kHz: $X_L = 2\pi fL = 2\pi(101.02\text{ kHz})(4.7\text{ mH}) = 2.98\text{ k}\Omega$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(101.02\text{ kHz})(560\text{ pF})} = 2.81\text{ k}\Omega$

$\mathbf{V}_o = \dfrac{160\ \Omega(1\text{ V} \angle 0°)}{172 + j2.98\text{ k}\Omega - j2.81\text{ k}\Omega} = \dfrac{160\text{ V} \angle 0°}{172 + j170}$

$= \dfrac{160\text{ V} \angle 0°}{241.83 \angle 44.66°} = \mathbf{0.66\ V\angle -44.66°}$

d. $f = f_s$: $V_{o_{max}} = \mathbf{0.93\ V}$

$f = f_1 = 95.19$ kHz, $V_o = 0.707(0.93\text{ V}) = \mathbf{0.66\ V}$

$f = f_2 = 101.02$ kHz, $V_o = 0.707(0.93\text{ V}) = \mathbf{0.66\ V}$

30. a. $f_p = \dfrac{1}{2\pi\sqrt{LC}}\sqrt{1 - \dfrac{R_\ell^2 C}{L}} \cong \mathbf{159.15\ kHz}$

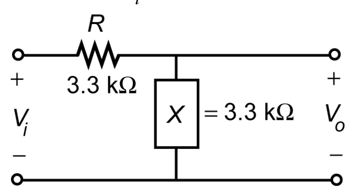
$$Q_\ell = \frac{X_L}{R_\ell} = \frac{2\pi f_p L}{R_\ell} = \frac{2\pi(159.15\text{ kHz})(1\text{ mH})}{16\,\Omega} = 62.5 \gg 10$$

$$Z_{T_p} = Q_\ell^2 R_\ell = (62.5)^2\ 16\ \Omega = 62.5\text{ k}\Omega \gg 4\text{ k}\Omega$$

and $V_o \cong V_i$ at resonance.

However, $R = 3.3\text{ k}\Omega$ affects the shape of the resonance curve and $BW = f_p/Q_\ell$ cannot be applied.

For $A_\upsilon = \dfrac{V_o}{V_i} = 0.707$, $|X| = R$ for the following configuration



For frequencies near f_p, $X_L \gg R_\ell$ and $\mathbf{Z}_L = R_\ell + jX_L \cong X_L$ and $X = X_L \parallel X_C$.

For frequencies near f_p but less than f_p

$$X = \frac{X_C X_L}{X_C - X_L}$$

and for $A_\upsilon = 0.707$

$$\frac{X_C X_L}{X_C - X_L} = R$$

Substituting $X_C = \dfrac{1}{2\pi f_1 C}$ and $X_L = 2\pi f_1 L$

the following equation can be derived:

$$f_1^2 + \frac{1}{2\pi RC} f_1 - \frac{1}{4\pi^2 LC} = 0$$

For this situation:

$$\frac{1}{2\pi RC} = \frac{1}{2\pi(3.3\text{ k}\Omega)(0.001\,\mu\text{F})} = 48.23 \times 10^3$$

$$\frac{1}{4\pi^2 LC} = \frac{1}{4\pi^2(1\text{ mH})(0.001\,\mu\text{F})} = 2.53 \times 10^{10}$$

and solving the quadratic equation, $f_1 = 135.83$ kHz

and $\dfrac{BW}{2} = f_p - f_1 =$ 159.15 kHz − 135.83 kHz = 22.32 kHz

so that $f_2 = f_p + \dfrac{BW}{2}$ = 159.15 kHz + 18.75 kHz = 177.9 kHz

b. $Q_p = \dfrac{f_p}{BW} = \dfrac{159.15\text{ kHz}}{44.64\text{ kHz}} = \mathbf{3.57}$

$BW = 2(18.75\text{ kHz}) = \mathbf{37.5\ kHz}$

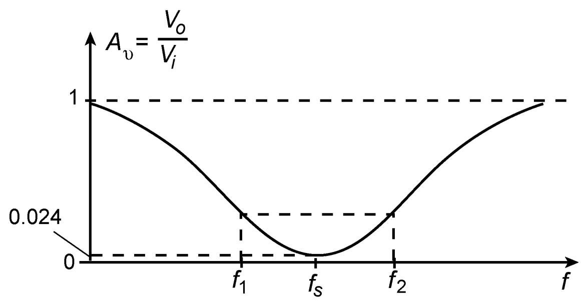
31. a. $Q_s = \dfrac{X_L}{R + R_\ell} = \dfrac{5000\,\Omega}{390\,\Omega + 10\,\Omega} = \dfrac{5000\,\Omega}{400\,\Omega} = \mathbf{12.5}$

b. $BW = \dfrac{f_s}{Q_s} = \dfrac{5000\text{ Hz}}{12.5} = \mathbf{400\ Hz}$

$f_1 = 5000\text{ Hz} - \dfrac{400\text{ Hz}}{2} = \mathbf{4.8\ kHz}$

$f_2 = 5000\text{ Hz} + \dfrac{400\text{ Hz}}{2} = \mathbf{5.20\ kHz}$

c.



At resonance

$V_o = \dfrac{10\,\Omega(V_i)}{10\,\Omega + 400\,\Omega}$

$= 0.024\ V_i$

d. At resonance, $10\,\Omega \parallel 2\text{ k}\Omega = 9.95\,\Omega$

$V_o = \dfrac{9.95\,\Omega(V_i)}{9.95\,\Omega + 400\,\Omega} \cong 0.024\ V_i$ as above!

32. a. $Q_\ell = \dfrac{X_L}{R_\ell} = \dfrac{400\,\Omega}{10\,\Omega} = 40$

$Z_{T_p} = Q_\ell^2 R_\ell = (40)^2\ 20\,\Omega = 32\text{ k}\Omega \gg 1\text{ k}\Omega$

At resonance, $V_o = \dfrac{32\text{ k}\Omega\, V_i}{32\text{ k}\Omega + 1\text{ k}\Omega} = 0.97 V_i$

and $A_\upsilon = \dfrac{V_o}{V_i} = 0.97$

For the low cutoff frequency note solution to Problem 30:

$$f_1^2 + \frac{1}{2\pi RC} f_1 - \frac{1}{4\pi^2 LC} = 0$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(20\text{ kHz})(400\,\Omega)} = 19.9\text{ nF}$$

$$L = \frac{X_L}{2\pi f} = \frac{400\,\Omega}{2\pi(20\text{ kHz})} = 3.18\text{ mH}$$

Substituting into the above equation and solving

$f_1 = 16.4\text{ kHz}$

with $\dfrac{BW}{2}$ = 20 kHz – 16.4 kHz = 3.6 kHz

and BW = 2(3.6 kHz) = **7.2 kHz**

$Q_p = \dfrac{f_p}{BW} = \dfrac{20\text{ kHz}}{7.2\text{ kHz}} = \mathbf{2.78}$

b. –

c. At resonance

Z_{T_p} = 32 kΩ ‖ 100 kΩ = 24.24 kΩ

with $V_o = \dfrac{24.24\text{ k}\Omega\, \text{V}_\text{i}}{24.24\text{ k}\Omega + 1\text{ k}\Omega} = 0.96V_i$

and $A_\upsilon = \dfrac{V_o}{V_i}$ = 0.96 vs 0.97 above

At frequencies to the right and left of f_p, the impedance Z_{T_p} will decrease and be affected less and less by the parallel 100 kΩ load. The characteristics, therefore, are only slightly affected by the 100 kΩ load.

d. At resonance

Z_{T_p} = 32 kΩ ‖ 20 kΩ = 12.31 kΩ

with $V_o = \dfrac{12.31\text{ k}\Omega\, V_i}{12.31\text{ k}\Omega + 1\text{ k}\Omega} = 0.925V_i$ vs 0.97 V_i above

At frequencies to the right and left of f_p, the impedance of each frequency will actually be less due to the parallel 20 kΩ load. The effect will be to narrow the resonance curve and decrease the bandwidth with an increase in Q_p.

33. a. $f_p = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(400\,\mu\text{H})(120\text{ pF})}}$ = **726.44 kHz** (band-stop)

$$X_{L_s}\angle 90° + \left(X_{L_p}\angle 90° \parallel X_C\angle -90°\right) = 0$$

$$jX_{L_s} + \frac{\left(X_{L_p}\angle 90°\right)\left(X_C\angle -90°\right)}{jX_{L_p} - jX_C} = 0$$

$$jX_{L_s} + \frac{X_{L_p}X_C}{j\left(X_{L_p} - X_C\right)} = 0$$

$$jX_{L_s} - j\frac{X_{L_p}X_C}{\left(X_{L_p} - X_C\right)} = 0$$

$$X_{L_s} - \frac{X_{L_p}X_C}{X_{L_p} - X_C} = 0$$

$$X_{L_s}X_C - X_{L_s}X_{L_p} + X_{L_p}X_C = 0$$

$$\frac{\omega L_s}{\omega C} - \omega L_s + \frac{\omega L_p}{\omega C} = 0$$

$$L_s L_p \omega^2 - \frac{1}{C}[L_s + L_p] = 0$$

$$\omega = \sqrt{\frac{L_s + L_p}{C L_s L_p}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{L_s + L_p}{C L_s L_p}} = \frac{1}{2\pi}\sqrt{\frac{460\times 10^{-6}}{28.8\times 10^{-19}}} = \mathbf{2.01\ MHz}\text{ (pass-band)}$$

34. a. $f_s = \dfrac{1}{2\pi\sqrt{LC}} \Rightarrow L_s = \dfrac{1}{4\pi^2 f_s^2 C} = \dfrac{1}{4\pi^2(100\text{ kHz})^2(200\text{ pF})} = 12.68\text{ mH}$

$X_L = 2\pi fL = 2\pi(30\text{ kHz})(12.68\text{ mH}) = 2388.91\ \Omega$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(30\text{ kHz})(200\text{ pF})} = 26.54\text{ k}\Omega$

$X_C - X_L = 26.54\text{ k}\Omega - 2388.91\ \Omega = 24.15\text{ k}\Omega(C)$

$X_{L_p} = X_{C_{(\text{net})}} = 24.15\text{ k}\Omega$

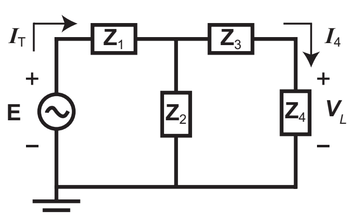
$L_p = \dfrac{X_L}{2\pi f} = \dfrac{24.15\text{ k}\Omega}{2\pi(30\text{ kHz})} = \mathbf{128.19\ mH}$

35. a. At low frequencies, L_1, $L_2 \Rightarrow$ short circuits and $C \Rightarrow$ open circuit. The result is $\mathbf{V}_L$ very close to $\mathbf{V}_i$ at low frequencies. At high frequencies, X_C shorts to ground and X_{L_2} has a high impedance so $\mathbf{V}_L$ approaches 0 V.

b. Determine fequency when $R_L = X_{L_2}$ due to voltage divide action.

$$X_L = R \Rightarrow 2\pi fL = R \Rightarrow f = \frac{R}{2\pi L} = \frac{220\ \Omega}{2\pi(0.22\text{ mH})} = 159.15\text{ kHz}$$

Since f_c less than split between X_L and R_L let us try 140 kHz.



$Z_1 = X_{L_1} = 2\pi fL_1 = 2\pi(140\text{ kHz})(0.47\text{ mH}) = 413.43\ \Omega$

$Z_2 = X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(140\text{ kHz})(5\text{ nF})} = 227.36\ \Omega$

$Z_3 = X_{L_2} = 2\pi fL_2 = 2\pi(140\text{ kHz})(0.22\text{ mH}) = 193.52\ \Omega$

$Z_4 = R = 220\ \Omega$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel (\mathbf{Z}_3 + \mathbf{Z}_4)$$

$$= j413.43\ \Omega + \frac{(227.36\ \Omega\angle -90^\circ)(220\ \Omega + j193.52\ \Omega)}{(-j227.36\ \Omega) + (220\ \Omega + j193.52\ \Omega)}$$

$$= j413.43\ \Omega + \frac{(227.36\ \Omega\angle -90^\circ)(293\ \Omega\angle 41.34^\circ)}{220\ \Omega - j33.84\ \Omega}$$

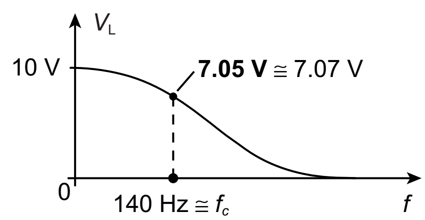
$$= j413.43\ \Omega + \frac{66.61\text{ k}\Omega\angle -48.66^\circ}{222.59\angle -8.74^\circ}$$

$$= j413.43\ \Omega + 299.25\ \Omega\angle -39.92^\circ$$

$= j413.43\ \Omega + 229.51\ \Omega - j192.03\ \Omega$
$= 229.51\ \Omega + j221.40\ \Omega$
$\mathbf{Z}_T = 318.89\ \Omega \angle 43.97°$

$$\mathbf{I}_T = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{10\text{ V} \angle 0°}{318.89\ \Omega \angle 43.97°} = 31.36\text{ mA} \angle{-43.97°}$$

$$\mathbf{I}_4 = \frac{\mathbf{Z}_2\mathbf{I}_T}{\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4} = \frac{(227.36\ \Omega \angle -90°)(31.36\text{ mA} \angle -43.97°)}{-j227.36\ \Omega + j193.52\ \Omega + 220\ \Omega}$$
$$= \frac{7.13\text{ A} \angle -133.97°}{220 - j33.84°} = \frac{7.13\text{ A} \angle -133.97°}{222.59 \angle -8.74°}$$
$= 32.03\text{ mA} \angle{-125.23°}$
$\mathbf{V}_L = \mathbf{I}_4\mathbf{Z}_4 = (32.03\text{ mA} \angle{-125.23°})(220\ \Omega \angle 0°)$
$= 7.05\text{ V} \angle{-125.23°}$



36. a. At very low frequencies $X_C \Rightarrow$ open-circuit and $X_L \Rightarrow$ short-circuit resulting in $\mathbf{V}_L \Rightarrow$ 0 V.

At very high frequencies $X_C \Rightarrow$ short-circuit and $X_L \Rightarrow$ open-circuit resulting in $\mathbf{V}_L \Rightarrow$ 20 V.

b. Utilize $X_{L_2} = X_C$ as a starting point to establish a frequency of application:

$$2\pi fL_2 = \frac{1}{2\pi fC} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(100\text{ mH})(0.12\ \mu\text{F})}} = 1.45\text{ kHz}$$

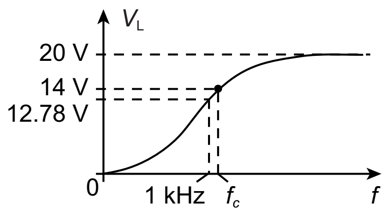
Try $f = 1$ kHz:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1\text{ kHz})(0.12\ \mu\text{F})} = 1.33\text{ k}\Omega$$
$X_{L_2} = 2\pi fL_2 = 2\pi(1\text{ kHz})(100\text{ mH}) = 628.32\ \Omega$
$\mathbf{Z}' = \mathbf{X}_{L_2} \parallel \mathbf{R}_L = (628.32\ \Omega \angle 90°) \parallel (1.2\text{ k}\Omega \angle 0°)$
$$= \frac{(628.32\ \Omega \angle 90°)(1.2\text{ k}\Omega \angle 0°)}{1.2\text{ k}\Omega + j628.32\ \Omega} = \frac{753.98 \times 10^3\ \Omega \angle 90°}{1.35 \times 10^3 \angle 27.64°}$$
$\mathbf{Z}' = 558.50\ \Omega \angle 62.36°$

$$\mathbf{V}_L = \frac{\mathbf{Z}' \cdot \mathbf{E}}{\mathbf{Z}' + \mathbf{X}_C} = \frac{(558.50\ \Omega \angle 62.36°)(20\text{ V} \angle 0°)}{558.50\ \Omega \angle 62.36° - j1.33\text{ k}\Omega}$$
$$= \frac{11.17 \times 10^3\text{ V} \angle 62.36°}{2.591 + j494.76 - j1.33\text{ k}\Omega}$$

$$= \frac{11.17\times10^3\ \text{V}\angle 62.36°}{2.591 - j835} = \frac{11.17\times10^3\ \text{V}\angle 62.36°}{874.28\angle -72.76°}$$

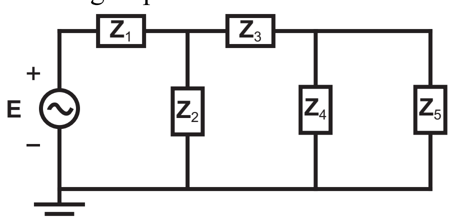
$\mathbf{V}_L = 12.78\ \text{V}\ \angle 135.12°$
which is very close to the cutoff value of 0.707(20 V) = 14.14 V. The cutoff frequency will be slightly higher than 1 kHz.



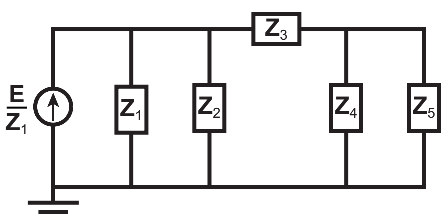
37. a. At very low frequencies $X_L \Rightarrow$ short-circuit and $X_C \Rightarrow$ open-circuit. The result is $\mathbf{V}_L \Rightarrow$ 60 V.

At very high frequencies $X_L \Rightarrow$ open-circuit and $X_C \Rightarrow$ short-circuit. The result is $\mathbf{V}_L \Rightarrow$ 0 V.

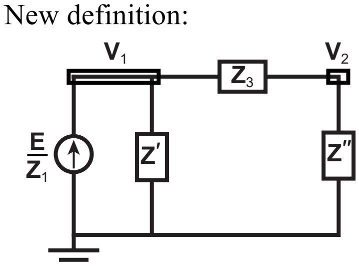
b. Defining impedances:



Source conversion:



New definition:



$\mathbf{Z}' = \mathbf{Z}_1 \parallel \mathbf{Z}_2$
$\mathbf{Z}'' = \mathbf{Z}_4 \parallel \mathbf{Z}_5$

Applying Nodal Analysis

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}'} + \frac{1}{\mathbf{Z}_3}\right] - \frac{1}{\mathbf{Z}_3}\mathbf{V}_2 = \frac{\mathbf{E}}{\mathbf{Z}_1}$$

$$\mathbf{V}_2\left[\frac{1}{\mathbf{Z}''}+\frac{1}{\mathbf{Z}_3}\right]-\frac{1}{\mathbf{Z}_3}\mathbf{V}_1 = 0$$

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}'}+\frac{1}{\mathbf{Z}_3}\right]-\frac{1}{\mathbf{Z}_3}\mathbf{V}_2 = \frac{\mathbf{E}}{\mathbf{Z}_1}$$

$$-\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right]+\left[\frac{1}{\mathbf{Z}''}+\frac{1}{\mathbf{Z}_3}\right]\mathbf{V}_2 = 0$$

$$\mathbf{V}_L = \mathbf{V}_2 = \frac{\begin{vmatrix}\frac{1}{\mathbf{Z}'}+\frac{1}{\mathbf{Z}_3} & \frac{\mathbf{E}}{\mathbf{Z}'}\\ -\frac{1}{\mathbf{Z}_3} & 0\end{vmatrix}}{\begin{vmatrix}\frac{1}{\mathbf{Z}'}+\frac{1}{\mathbf{Z}_3} & -\frac{1}{\mathbf{Z}_3}\\ -\frac{1}{\mathbf{Z}_3} & \frac{1}{\mathbf{Z}''}+\frac{1}{\mathbf{Z}_3}\end{vmatrix}} = \frac{\left[\frac{1}{\mathbf{Z}'}+\frac{1}{\mathbf{Z}_3}\right][0]-\left[\frac{\mathbf{E}}{\mathbf{Z}_1}\right]\left[-\frac{1}{\mathbf{Z}_3}\right]}{\left[\frac{1}{\mathbf{Z}'}+\frac{1}{\mathbf{Z}_3}\right]\left[\frac{1}{\mathbf{Z}''}+\frac{1}{\mathbf{Z}_3}\right]-\left[-\frac{1}{\mathbf{Z}_3}\right]\left[-\frac{1}{\mathbf{Z}_3}\right]}$$

$$= \frac{\mathbf{E}/\mathbf{Z}_1\mathbf{Z}_3}{\left[\frac{1}{\mathbf{Z}'}+\frac{1}{\mathbf{Z}_3}\right]\left[\frac{1}{\mathbf{Z}''}+\frac{1}{\mathbf{Z}_3}\right]-\frac{1}{\mathbf{Z}_3^2}}$$

Choosing $X_C = \frac{1}{3}X_L$ to bring $\mathbf{V}_L$ down to 0.707 level

$$\frac{1}{2\pi fC} = \frac{1}{3}2\pi fL \text{ and } f = \frac{\sqrt{3}}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1\text{ mH})(5\text{ nF})}} = 123.13\text{ kHz}$$

Using f = **120 kHz**

$\mathbf{Z}_1$: $X_L = 2\pi fL = 2\pi(120\text{ kHz})(1\text{ mH}) = 754\ \Omega\ \angle 90°$

$\mathbf{Z}_2$: $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(120\text{ kHz})(5\text{ nF})} = 265\ \Omega\ \angle -90°$

$\mathbf{Z}_3$: $754\ \Omega\ \angle 90°$

$\mathbf{Z}_4$: $265\ \Omega\ \angle -90°$

$\mathbf{Z}_5$: $2.2\text{ k}\Omega\ \angle 0°$

$\mathbf{Z}_1\cdot\mathbf{Z}_3 = (754\ \Omega\ \angle 90°)(754\ \Omega\ \angle 90°) = 568.52\times 10^3\ \Omega^2\ \angle 180°$

$$\frac{\mathbf{E}}{\mathbf{Z}_1\mathbf{Z}_3} = \frac{60\text{ V}\angle 0°}{568.52\times 10^3\ \Omega^2\angle 180°} = 105.5\times 10^{-6}\ \angle -180°$$

$$\mathbf{Z}' = \mathbf{Z}_1 \parallel \mathbf{Z}_2 = \frac{(754\ \Omega\angle 90°)(265\ \Omega\angle -90°)}{+j754\ \Omega - j265\ \Omega}$$

$$= \frac{199.81\times 10^3\ \Omega\angle 0°}{489\angle 90°}$$

$$= 408.61\ \Omega\ \angle -90°$$

$$\mathbf{Z}'' = \mathbf{Z}_4 \parallel \mathbf{Z}_5 = \frac{(265\ \Omega \angle -90°)(2.2\ \text{k}\Omega \angle 0°)}{2.2\ \text{k}\Omega - j265\ \Omega} = \frac{538\times 10^3 \Omega \angle -90°}{2.2 \angle -6.87°}$$
$$= 262.61\ \Omega \angle -83.13°$$

$$\left[\frac{1}{\mathbf{Z}'} + \frac{1}{\mathbf{Z}_3}\right] = \left[\frac{1}{408.61\ \Omega \angle -90°} + \frac{1}{754\ \Omega \angle 90°}\right]$$
$$= 2.44 \times 10^{-3} \angle 90° + 1.33 \times 10^{-3} \angle -90°$$
$$= j2.44 \times 10^{-3} - j1.33 \times 10^{-3}$$
$$= j1.11 \times 10^{-3}$$

$$\left[\frac{1}{\mathbf{Z}''} + \frac{1}{\mathbf{Z}_3}\right] = \frac{1}{262.61 \angle -83.13°} + \frac{1}{754\ \Omega \angle 90°}$$
$$= 3.81 \times 10^{-3} \angle 83.13° + 1.33 \times 10^{-3} \angle -90°$$
$$= 0.455 \times 10^{-3} + j3.78 \times 10^{-3} - j1.33 \times 10^{-3}$$
$$= 0.455 \times 10^{-3} + j2.54 \times 10^{-3}$$
$$= 2.49 \times 10^{-3} \angle 79.48°$$

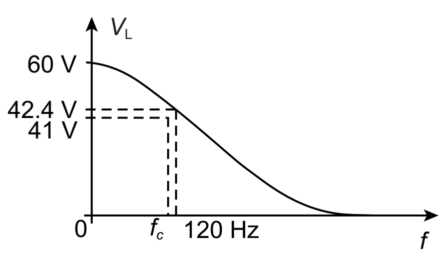
$$[1.11 \times 10^{-3} \angle 90°][2.49 \times 10^{-3} \angle 79.48°]$$
$$= 2.76 \times 10^{-6} \angle 169.48°$$

$$\mathbf{Z}_3^2 = (754\ \Omega \angle 90°)(754\ \Omega \angle 90°)$$
$$= 5.69 \times 10^{-6} \angle 180°$$
$$\frac{1}{\mathbf{Z}_3^2} = \frac{1}{5.69\times 10^6 \angle 180°} = 0.176 \times 10^{-6} \angle -180°$$

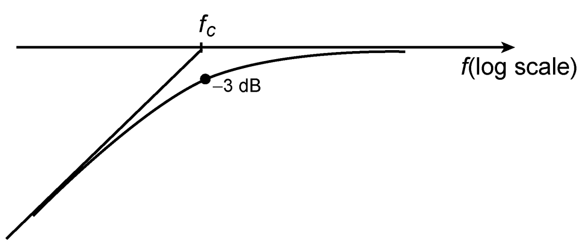
---

and

$$\mathbf{V}_L = \frac{105.5\times 10^6 \angle -180°}{2.58\times 10^6 \angle 168.8°}$$
$$= 40.89\ \text{V} \angle -11.2°$$
$\cong$ 41 V which is very close to 0.707(60 V) = 42.42 V



38. a, b. $f_c = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi(0.47\ \text{k}\Omega)(0.047\ \mu\text{F})}$ = **7.2 kHz**



c. $f = \frac{1}{2}f_c$: $A_{v_{dB}} = 20\log_{10}\frac{1}{\sqrt{1+(f_c/f)^2}} = 20\log_{10}\frac{1}{\sqrt{1+(2)^2}} = \mathbf{-7\ dB}$

$f = 2f_c$: $A_{v_{dB}} = 20\log_{10}\frac{1}{\sqrt{1+(0.5)^2}} = \mathbf{-0.969\ dB}$

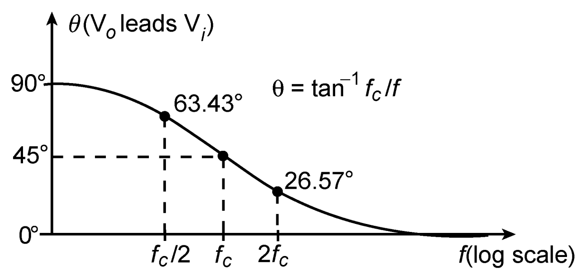
$f = \frac{1}{10}f_c$: $A_{v_{dB}} = 20\log_{10}\frac{1}{\sqrt{1+(10)^2}} = \mathbf{-20.04\ dB}$

$f = 10f_c$: $A_{v_{dB}} = 20\log_{10}\frac{1}{\sqrt{1+(0.1)^2}} = \mathbf{-0.043\ dB}$

d. $f = \frac{1}{2}f_c$: $A_v = \frac{1}{\sqrt{1+(f_c/f)^2}} = \frac{1}{\sqrt{1+(2)^2}} = \mathbf{0.447}$

$f = 2f_c$: $A_v = \frac{1}{\sqrt{1+(0.5)^2}} = \mathbf{0.894}$

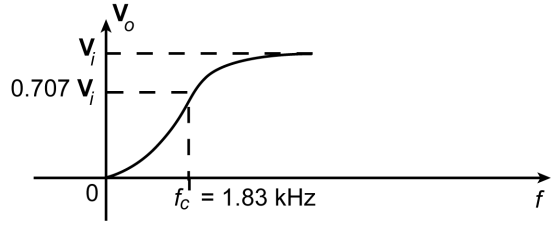
e.



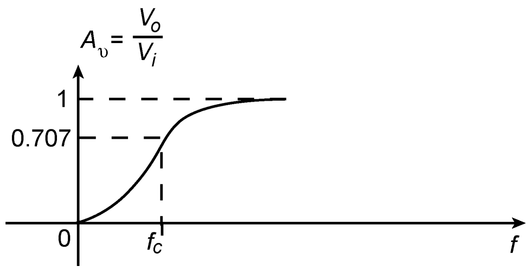
39. a. $f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(6.8\text{ k}\Omega \parallel 12\text{ k}\Omega)0.02\ \mu\text{F}} = \frac{1}{2\pi(4.34\text{ k}\Omega)(0.02\ \mu\text{F})} = \mathbf{1.83\ kHz}$

$\frac{V_o}{V_i} = \frac{1}{\sqrt{1+(f_c/f)^2}}$

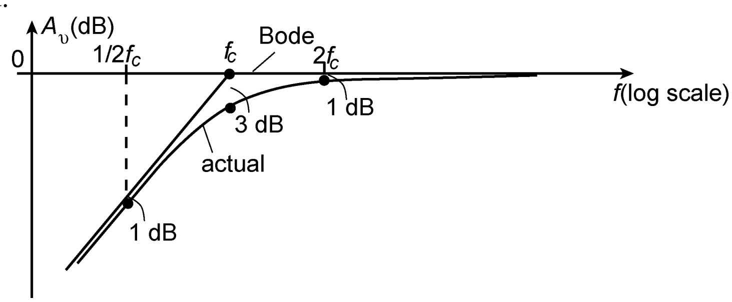
and $V_o = \left(\frac{1}{\sqrt{1+(f_c/f)^2}}\right)V_i$



b.



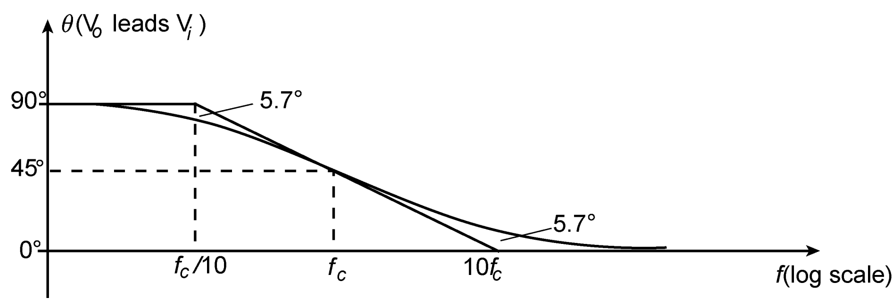
c. & d.



e. Remember the log scale! $1.5f_c$ is not midway between f_c and $2f_c$

$$A_{\upsilon_{\text{dB}}} = 20 \log_{10} A_\upsilon$$
$$-1.5 = 20 \log_{10} A_\upsilon$$
$$-0.075 = \log_{10} A_\upsilon$$
$$A_\upsilon = \frac{V_o}{V_i} = \mathbf{0.84}$$

f. $\theta = \tan^{-1} f_c/f$



40. a, b. $\mathbf{A}_\upsilon = \dfrac{\mathbf{V}_o}{\mathbf{V}_i} = A_\upsilon \angle\theta = \dfrac{1}{\sqrt{1 + (f/f_c)^2}} \angle{-\tan^{-1} f/f_c}$

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(12\ \text{k}\Omega)(1000\ \text{pF})} = \mathbf{13.26\ kHz}$$

c. $f = f_c/2 = 6.63$ kHz

$$A_{v_{\mathrm{dB}}} = 20\log_{10}\frac{1}{\sqrt{1+(0.5)^2}} = \mathbf{-0.97\ dB}$$

$f = 2f_c = 26.52$ kHz

$$A_{v_{\mathrm{dB}}} = 20\log_{10}\frac{1}{\sqrt{1+(2)^2}} = \mathbf{-6.99\ dB}$$

$f = f_c/10 = 1.326$ kHz

$$A_{v_{\mathrm{dB}}} = 20\log_{10}\frac{1}{\sqrt{1+(0.1)^2}} = \mathbf{-0.04\ dB}$$

$f = 10f_c = 132.6$ kHz

$$A_{v_{\mathrm{dB}}} = 20\log_{10}\frac{1}{\sqrt{1+(10)^2}} = \mathbf{-20.04\ dB}$$

d. $f = f_c/2$: $A_v = \dfrac{1}{\sqrt{1+(0.5)^2}} = \mathbf{0.894}$

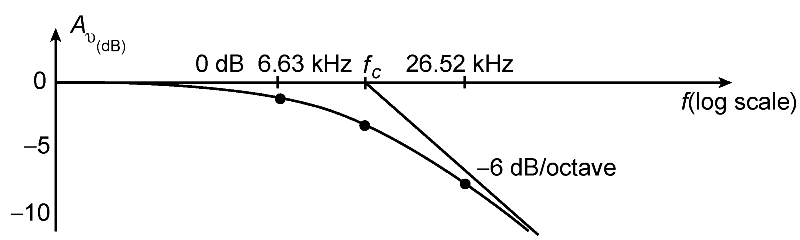
$f = 2f_c$: $A_v = \dfrac{1}{\sqrt{1+(2)^2}} = \mathbf{0.447}$

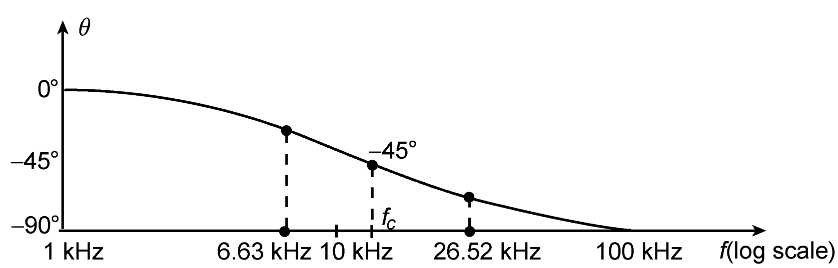
e. $\theta = \tan^{-1} f/f_c$

$f = f_c/2$: $\theta = -\tan^{-1} 0.5 = -26.57°$

$f = f_c$: $\theta = -\tan^{-1} 1 = -45°$

$f = 2f_c$: $\theta = -\tan^{-1} 2 = -63.43°$





41. a. $R_2 \parallel X_C = \dfrac{(R_2)(-jX_C)}{R_2 - jX_C} = -j\dfrac{R_2 X_C}{R_2 - jX_C}$

$$\mathbf{V}_o = \frac{\left(\dfrac{-jR_2X_C}{R_2 - jX_C}\right)\mathbf{V}_i}{R_1 - j\dfrac{R_2X_C}{R_2 - jX_C}} = -j\frac{R_2X_C\mathbf{V}_i}{R_1(R_2 - jX_C) - jR_2X_C}$$

$$= \frac{-jR_2X_C\mathbf{V}_i}{R_1R_2 - jR_1X_C - jR_2X_C} = \frac{-jR_2X_C\mathbf{V}_i}{R_1R_2 - j(R_1+R_2)X_C}$$

$$= \frac{R_2X_C\mathbf{V}_i}{jR_1R_2 + (R_1+R_2)X_C} = \frac{R_2\mathbf{V}_i}{j\dfrac{R_1R_2}{X_C} + (R_1+R_2)}$$

$$= \frac{R_2\mathbf{V}_i}{R_1 + R_2 + j\dfrac{R_1R_2}{X_C}} = \frac{\left(\dfrac{R_2}{R_1+R_2}\right)\mathbf{V}_i}{1 + j\left(\dfrac{R_1R_2}{R_1+R_2}\right)\dfrac{1}{X_C}}$$

and $\mathbf{A}_\upsilon = \dfrac{\mathbf{V}_o}{\mathbf{V}_i} = \dfrac{\dfrac{R_2}{R_1+R_2}}{1 + j\omega\left(\dfrac{R_1R_2}{R_1+R_2}\right)C}$

or $\mathbf{A}_\upsilon = \dfrac{R_2}{R_1+R_2}\left[\dfrac{1}{1 + j2\pi f(R_1 \| R_2)C}\right]$

defining $f_c = \dfrac{1}{2\pi(R_1 \| R_2)C}$

$\mathbf{A}_\upsilon = \dfrac{R_2}{R_1+R_2}\left[\dfrac{1}{1 + jf/f_c}\right]$

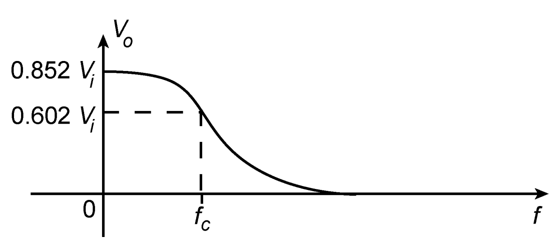
and $\mathbf{A}_\upsilon = \dfrac{R_2}{R_1+R_2}\left[\dfrac{1}{\sqrt{1+(f/f_c)^2}} \angle -\tan^{-1} f/f_c\right]$

with $|V_o| = \dfrac{R_2}{R_1+R_2}\left[\dfrac{1}{\sqrt{1+(f/f_c)^2}}\right]|V_i|$

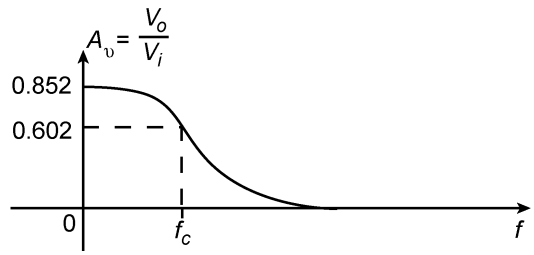
for $f \ll f_c$, $V_o = \dfrac{R_2}{R_1+R_2}V_i = \dfrac{27\text{ k}\Omega}{4.7\text{ k}\Omega + 27\text{ k}\Omega}V_i = 0.852V_i$

at $f = f_c$: $V_o = 0.852[0.707]V_i = 0.602V_i$

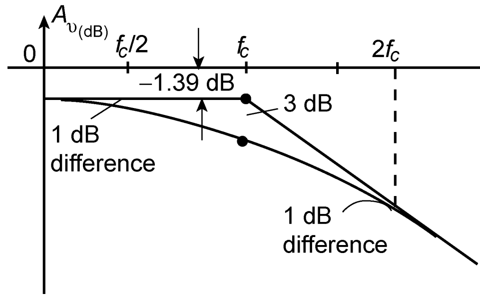
$f_c = \dfrac{1}{2\pi(R_1 \| R_2)C} = \dfrac{1}{2\pi(4.7\text{ k}\Omega \| 27\text{ k}\Omega)0.039\ \mu\text{F}} =$ **1.02 kHz**



b.



c. & d.



$$-20\log_{10}\frac{R_1+R_2}{R_2} = -20\log_{10}\frac{4.7\text{ k}\Omega + 27\text{ k}\Omega}{27\text{ k}\Omega}$$

$$= -20\log_{10} 1.174 = -1.39\text{ dB}$$

e. $A_{\upsilon_{dB}} \cong -1.39\text{ dB} - 0.5\text{ dB} = -1.89\text{ dB}$

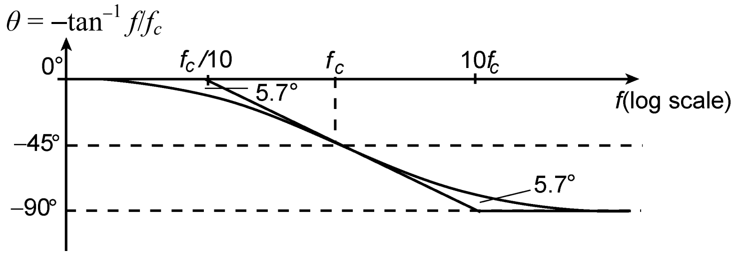
$$A_{\upsilon_{dB}} = 20\log_{10} A_\upsilon$$

$$-1.89 = 20\log_{10} A_\upsilon$$

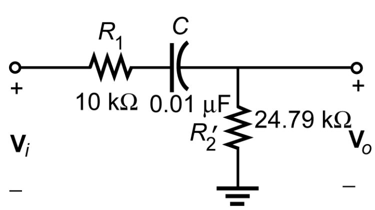
$$0.0945 = \log_{10} A_\upsilon$$

$$A_\upsilon = \frac{V_o}{V_i} = \mathbf{0.80}$$

f. $\theta = -\tan^{-1} f/f_c$



42.



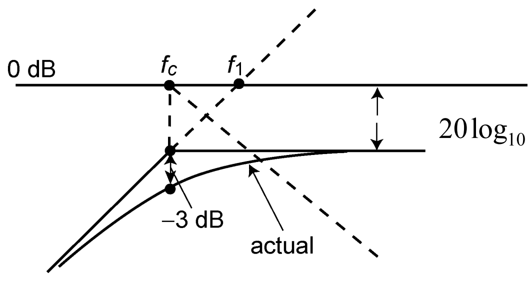
$R_2' = 39\text{ k}\Omega \parallel 68\text{ k}\Omega = 24.79\text{ k}\Omega$

a. From Section 21.11,

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{jf/f_1}{1 + jf/f_c}$$

$$f_1 = \frac{1}{2\pi R_2' C} = \frac{1}{2\pi(24.79\ \text{k}\Omega)(0.01\,\mu\text{F})} = \mathbf{642.01\ Hz}$$

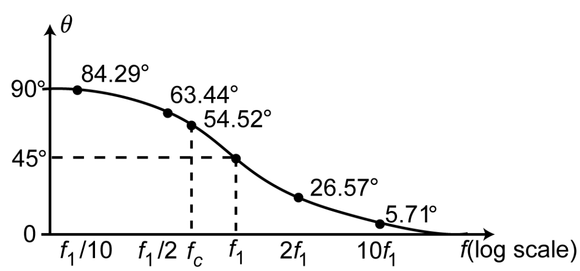
$$f_c = \frac{1}{2\pi\left(R_1 + R_2'\right)C} = \frac{1}{2\pi(10\ \text{k}\Omega + 24.79\ \text{k}\Omega)(0.01\,\mu\text{F})} = \mathbf{457.47\ Hz}$$



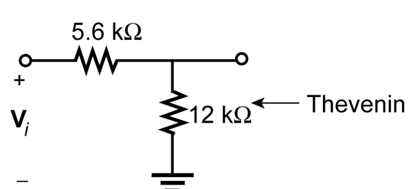
$$20\log_{10}\frac{f}{f_1} = 20\log_{10}\frac{f_c}{f_1}$$
$$= 20\ \log_{10}\frac{457.5\ \text{Hz}}{642\ \text{Hz}}$$
$$= \mathbf{-2.94\ dB}$$

b. $\theta = 90° - \tan^{-1}\dfrac{f}{f_1} = +\tan^{-1}\dfrac{f_1}{f}$

$f = f_1$: $\theta = 45°$
$f = f_c$: $\theta = 54.52°$
$f = \dfrac{1}{2}f_1 = 321$ Hz, $\theta = 63.44°$
$f = \dfrac{1}{10}f_1 = 64.2$ Hz, $\theta = 84.29°$
$f = 2f_1 = 1{,}284$ Hz, $\theta = 26.57°$
$f = 10f_1 = 6420$ Hz, $\theta = 5.71°$

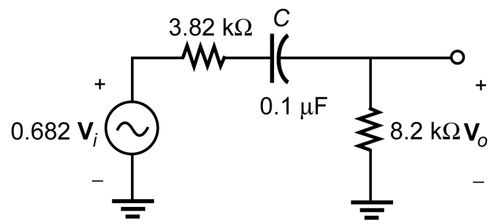


43. a.



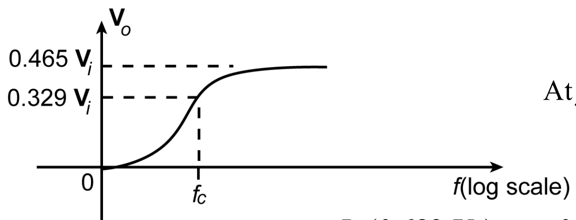
$$V_{Th} = \frac{12\ \text{k}\Omega\ V_i}{12\ \text{k}\Omega + 5.6\ \text{k}\Omega} = 0.682\ V_i$$

$R_{Th} = 5.6\ \text{k}\Omega \parallel 12\ \text{k}\Omega = 3.82\ \text{k}\Omega$



$f = \infty$ Hz: ($C \Rightarrow$ short circuit)

$$\mathbf{V}_o = \frac{8.2\text{ k}\Omega\,(0.682\,\mathbf{V}_i)}{8.2\text{ k}\Omega + 3.82\text{ k}\Omega} = 0.465\,\mathbf{V}_i$$



At f_c: $\mathbf{V}_0 = 0.707(0.465\,\mathbf{V}_i) = 0.329\,\mathbf{V}_i$

voltage-divider rule: $\mathbf{V}_o = \dfrac{R_2(0.682\,\mathbf{V}_i)}{R_1 + R_2 - jX_C} = \dfrac{0.682\,R_2\mathbf{V}_i}{R_1 + R_2 - jX_C}$

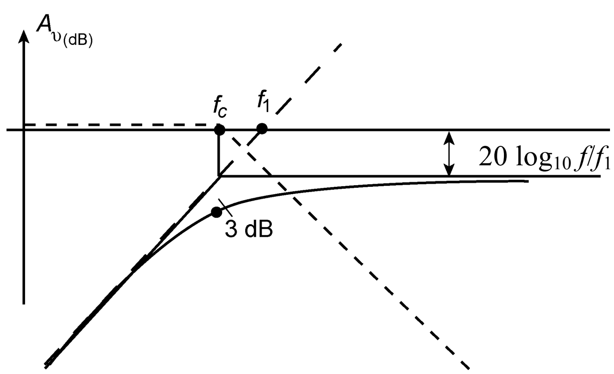
and $\mathbf{A}_\upsilon = \dfrac{\mathbf{V}_o}{\mathbf{V}_i} = \dfrac{0.682\,R_2}{R_1 + R_2 - jX_C} = \dfrac{j2\pi f(0.682\,R_2)C}{1 + j2\pi f(R_1 + R_2)C}$

so that $\mathbf{A}_\upsilon = \dfrac{jf/f_1}{1 + jf/f_c}$ with $f_1 = \dfrac{1}{2\pi 0.682 R_2 C} = \dfrac{1}{2\pi 0.682(8.2\text{ k}\Omega)(0.1\,\mu\text{F})}$

$= 284.59$ Hz

and $f_c = \dfrac{1}{2\pi(R_1 + R_2)C} = \dfrac{1}{2\pi(3.82\text{ k}\Omega + 8.2\text{ k}\Omega)(0.1\,\mu\text{F})}$

$=$ **132.41 Hz**

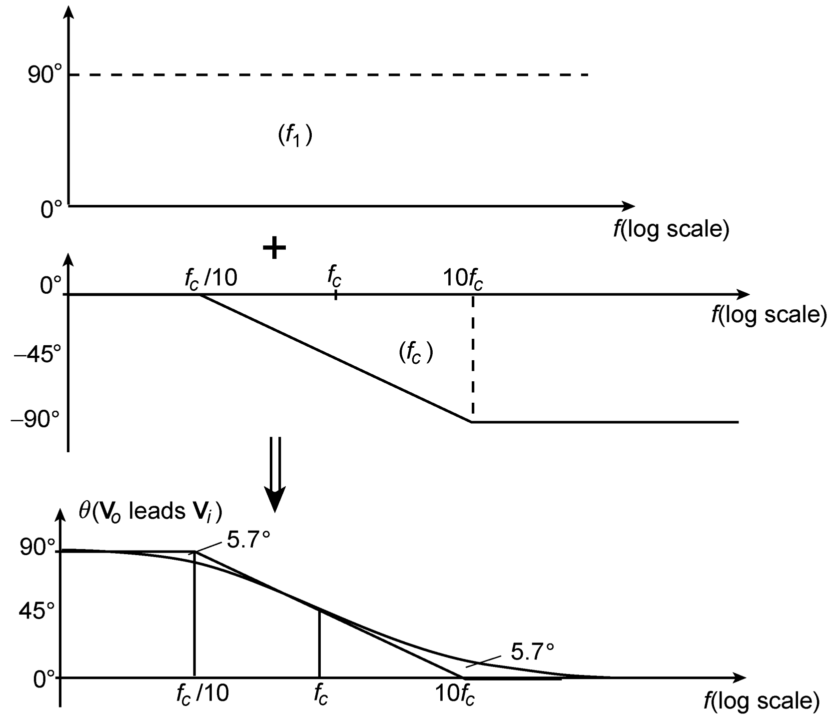


$20\log_{10} f/f_1 = 20\log_{10}\dfrac{132.41\text{ Hz}}{284.59\text{ Hz}}$

$= 20\log_{10} 0.465 =$ **−6.65 dB**

b. $\theta = 90° - \tan^{-1} f/f_c = +\tan^{-1} f_c/f = \tan^{-1} 132.6\text{ Hz}/f$

or

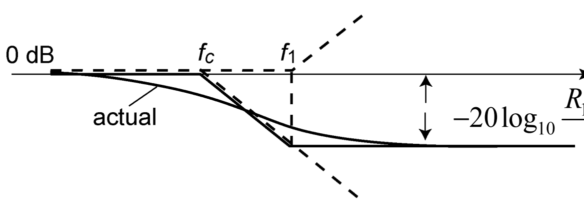


44. a. $\mathbf{A}_v = \dfrac{1 + j\dfrac{f}{f_1}}{1 + j\dfrac{f}{f_c}}$

$f_1 = \dfrac{1}{2\pi R_2 C} = \dfrac{1}{2\pi(10\ \text{k}\Omega)(800\ \text{pF})} = 19.89\ \text{kHz}$

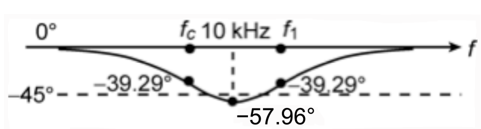
$f_c = \dfrac{1}{2\pi(R_1 + R_2)C} = \dfrac{1}{2\pi(10\ \text{k}\Omega + 91\ \text{k}\Omega)(800\ \text{pF})}$

$= 1.97\ \text{kHz}$



$-20\log_{10}\dfrac{R_1 + R_2}{R_2} = -20\log_{10}10 = \mathbf{-20\ dB}$

b.



$\theta = \tan^{-1} f/f_1 - \tan^{-1} f/f_c$

$f = 10$ kHz

$\theta = \tan^{-1}\dfrac{10\ \text{kHz}}{19.89\ \text{kHz}} - \tan^{-1}\dfrac{10\ \text{kHz}}{1.97\ \text{kHz}} = 29.66° - 87.62° = \mathbf{-57.96°}$

$f = f_c$: $(f_1 = 10\,f_c)$

$\theta = \tan^{-1}\dfrac{f_c}{10\,f_c} - \tan^{-1}\dfrac{f_c}{f_c} = \tan^{-1}0.1 \tan^{-1}1 = 5.71° - 45° = \mathbf{-39.29°}$

45. a. R_1 no effect!
Note Section 22.12.

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1 + j(f/f_1)}{1 + j(f/f_c)}$$

$$f_1 = \frac{1}{2\pi(5.6\text{ k}\Omega)(0.01\,\mu\text{F})} = 2.84\text{ kHz}$$

$$f_c = \frac{1}{2\pi(12\text{ k}\Omega + 5.6\text{ k}\Omega)(0.01\,\mu\text{F})} = 904.3\text{ Hz}$$

Note Fig. 22.65.

Asymptote at 0 dB from $0 \rightarrow f_c$
−6 dB/octave from f_c to f_1
−9.95 dB from f_1 on $\left(-20 \log \frac{12\text{ k}\Omega + 5.6\text{ k}\Omega}{5.6\text{ k}\Omega} = -9.5\text{ dB}\right)$

(b) Note Fig. 22.67.

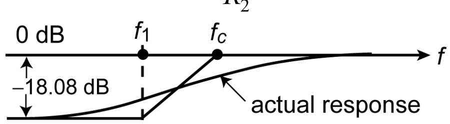
From 0° to −26.50° at f_c and f_1
$\theta = \tan^{-1} f/f_1 - \tan^{-1} f/f_c$
At $f = 1500$ Hz (between f_c and f_1)
$\theta = \tan^{-1} 1500\text{ Hz}/2.84\text{ kHz} - \tan^{-1} 1500\text{ Hz}/904.3\text{ Hz}$
$= 27.83° - 58.92° = -31.09°$

46. a. $$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1 - jf_1/f}{1 - jf_c/f}$$

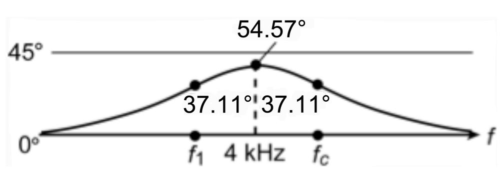
$$f_1 = \frac{1}{2\pi R_1 C} = \frac{1}{2\pi(3.3\text{ k}\Omega)(0.05\,\mu\text{F})} = 964.58\text{ Hz}$$

$$f_c = \frac{1}{2\pi(R_1 \| R_2)C} = \frac{1}{2\pi\underbrace{(3.3\text{ k}\Omega \| 0.47\text{ k}\Omega)}_{0.411\text{ k}\Omega}(0.05\,\mu\text{F})} = \mathbf{7.74\text{ kHz}}$$

$$-20\log_{10}\frac{R_1 + R_2}{R_2} = -20\log_{10}\frac{3.3\text{ k}\Omega + 0.47\text{ k}\Omega}{0.47\text{ k}\Omega} = -20\log_{10} 8.02 = -18.08\text{ dB}$$



b.



$$\theta = -\tan^{-1}\frac{f_1}{f} + \tan^{-1}\frac{f_c}{f}$$

$$f = f_1 : \theta = -\tan^{-1}\frac{f_1}{f_1} + \tan^{-1}\frac{f_c}{f_1}$$

$$= -\tan^{-1} 1 + \tan^{-1}\frac{7.74\text{ kHz}}{964.58\text{ Hz}}$$

$$= -45° + 92.1° = \mathbf{47.1°}$$

$$f = 4\text{ kHz} : \theta = -\tan^{-1}\frac{964.58\text{ Hz}}{4\text{ kHz}} + \tan^{-1}\frac{7.74\text{ kHz}}{4\text{ kHz}}$$

$$= -15.06° + 69.63° = \mathbf{54.57°}$$

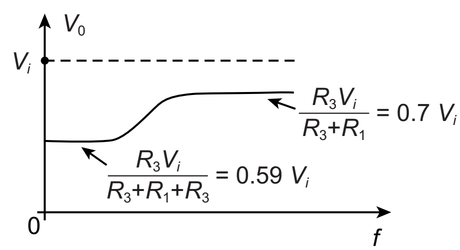
$$f = f_c : \theta = -\tan^{-1}\frac{964.58\text{ Hz}}{7.74\text{ kHz}} + \tan^{-1}\frac{f_c}{f_c}$$

$$= -\tan^{-1} 0.1246 + \tan^{-1} 1$$

$$= -7.89° + 45° = \mathbf{37.11°}$$

47. $f = 0\text{ Hz}: V_o = \dfrac{R_3 V_i}{R_3 + R_1 + R_2} = \dfrac{4.7\text{ k}\Omega\ V_i}{4.7\text{ k}\Omega + 2\text{ k}\Omega + 1.2\text{ k}\Omega} = 0.59\ V_i$

$f = \text{high}: V_o = \dfrac{R_3 V_i}{R_3 + R_1} = \dfrac{4.7\text{ k}\Omega\ V_i}{4.7\text{ k}\Omega + 2\text{ k}\Omega} = 0.7\ V_i$



$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_3}{R_3 + R_1 + R_2 \parallel X_C \angle -90°}$$

Define $R' = R_3 + R_1$

$$\text{and } \mathbf{A}_v = \frac{R_3}{R' + R_2 \parallel X_C \angle -90°} = \frac{R_3}{R' + \dfrac{R_2(-jX_C)}{R_2 - jX_C}}$$

$$= \frac{R_3(R_2 - jX_C)}{R'(R_2 - jX_C) - jR_2X_C}$$

$$= \frac{R_2R_3 - jR_3X_C}{R'R_2 - jR'X_C - jR_2X_C}$$

$$= \frac{R_2R_3 - jR_3X_C}{R'R_2 - j(R'+R_2)X_C} = \frac{1 - j\dfrac{R_3 X_C}{R_2 R_3}}{\dfrac{R'R_2}{R_2R_3} - j\dfrac{(R'+R_2)}{R_2R_3}X_C}$$

$$= \frac{1 - j\dfrac{X_C}{R_2}}{\dfrac{R'}{R_3}\left[1 - j\dfrac{(R'+R_2)}{R_2R_3}\dfrac{R_3}{R'}X_C\right]}$$

$$= \frac{1 - j\dfrac{X_C}{R_2}}{\dfrac{R'}{R_3}\left[1 - j\dfrac{R'+R_2}{R'R_2}X_C\right]}$$

$$= \frac{R_3}{R'}\frac{[1 - jX_C/R_2]}{\left[1 - j\left(\dfrac{R'+R_2}{R'R_2}\right)X_C\right]}$$

$$= \frac{R_3}{R_1+R_3}\frac{\left[1 - j\dfrac{1}{2\pi R_2C}\right]}{\left[1 - j\dfrac{1}{2\pi(R' \,||\, R_2)C}\right]}$$

and $\mathbf{A}_\upsilon = \dfrac{R_3}{R_1+R_3}\dfrac{[1 - j(f_1/f)]}{[1 - j(f_c/f)]}$ $\quad f_1 = \dfrac{1}{2\pi R_2 C}$, $\quad f_c = \dfrac{1}{2\pi(R' \,||\, R_2)C}$

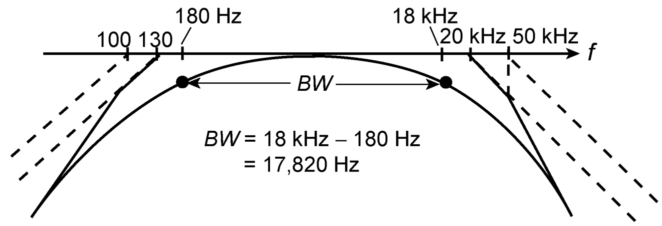
$$A_{\upsilon_{dB}} = 20\log_{10}\frac{R_3}{R_1+R_3} + 20\log_{10}\sqrt{1+(f_1/f)^2} + 20\log_{10}\frac{1}{\sqrt{1+(f_c/f)^2}}$$

$$= -20\log_{10}\frac{R_1+R_3}{R_3} + 20\log_{10}\sqrt{1+(f_1/f)^2} + 20\log_{10}\frac{1}{\sqrt{1+(f_c/f)^2}}$$

with $f_1 = \dfrac{1}{2\pi R_2C}$, $f_c = \dfrac{1}{2\pi(R' \,||\, R_2)C}$, $R' = R_1 + R_3$

$\theta = -\tan^{-1}(f_1/f) + \tan^{-1}(f_c/f)$

48. a. $$\frac{A_\upsilon}{A_{\upsilon\max}} = \frac{1}{\left(1 - j\dfrac{100\text{ Hz}}{f}\right)\left(1 - j\dfrac{130\text{ Hz}}{f}\right)\left(1 + j\dfrac{f}{20\text{ kHz}}\right)\left(1 + j\dfrac{f}{50\text{ kHz}}\right)}$$



Proximity of 100 Hz to 130 Hz will raise lower cutoff frequency above 130 Hz:

Testing: f = 180 Hz: (with lower terms only)

$$A_{v_{dB}} = -20\log_{10}\sqrt{1+\left(\frac{100}{f}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{130}{f}\right)^2}$$

$$= -20\log_{10}\sqrt{1+\left(\frac{100}{180}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{130}{180}\right)^2}$$

$$= 1.17\text{ dB} - 1.82\text{ dB} = \mathbf{-2.99\ dB} \cong \mathbf{-3\ dB}$$

Proximity of 50 kHz to 20 kHz will lower high cutoff frequency below 20 kHz:

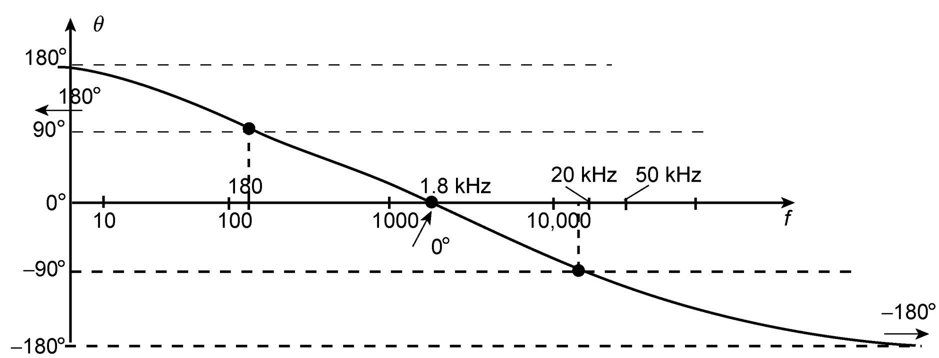
Testing: f = 18 kHz: (with upper terms only)

$$A_{v_{dB}} = -20\log_{10}\sqrt{1+\left(\frac{f}{20\text{ kHz}}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{f}{50\text{ kHz}}\right)^2}$$

$$= -20\log_{10}\sqrt{1+\left(\frac{18\text{ kHz}}{20\text{ kHz}}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{13\text{ kHz}}{20\text{ kHz}}\right)^2}$$

$$= -2.576\text{ dB} - 0.529\text{ dB} = \mathbf{-3.105\ dB}$$

b.



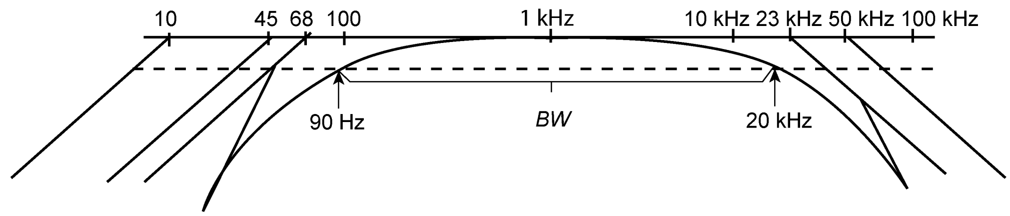
Testing: f = 1.8 kHz:

$$\theta = \tan^{-1}\frac{100}{1.8\text{ kHz}} + \tan^{-1}\frac{130}{1.8\text{ kHz}} - \tan^{-1}\frac{1.8\text{ kHz}}{20\text{ kHz}} - \tan^{-1}\frac{1.8\text{ kHz}}{50\text{ kHz}}$$

$$= 3.18° + 4.14° - 5.14° - 2.06°$$

$$= \mathbf{0.12°} \cong \mathbf{0°}$$

49.



50 kHz vs 23 kHz → drop about 1 dB at 23 kHz due to 50 kHz break.
Ignore effect of break frequency at 10 Hz.
Assume –2 dB drop at 68 Hz due to break frequency at 45 Hz.
Rough sketch suggests low cut-off frequency of 90 Hz.
Checking: Ignoring upper terms and using f = 90 Hz:

$$A'_{\upsilon dB} = -20\log_{10}\sqrt{1+\left(\frac{10\text{ Hz}}{f}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{45\text{ Hz}}{f}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{68\text{ Hz}}{f}\right)^2}$$

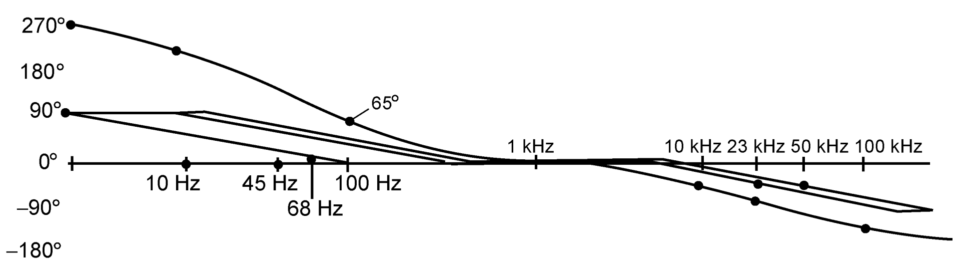
$= -0.0532\text{ dB} - 0.969\text{ dB} - 1.96\text{ dB}$
$= -2.98\text{ dB}$ (excellent)

High frequency cutoff: Try f = 20 kHz

$$A'_{\upsilon dB} = -20\log_{10}\sqrt{1+\left(\frac{f}{23\text{ kHz}}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{f}{50\text{ kHz}}\right)^2}$$

$= -2.445\text{ dB} - 0.6445\text{ dB}$
$= -3.09\text{ dB}$ (excellent

∴ BW = 20 kHz – 90 Hz = **19,910 Hz** ≅ 20 kHz
f_1 = **90 Hz**, f_2 = **20 kHz**



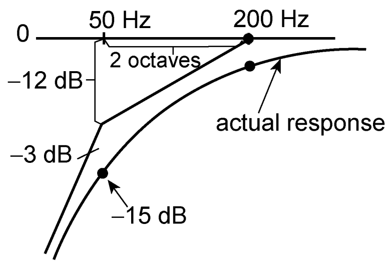
Testing: f = 100 Hz

$$\theta = \tan^{-1}\frac{10\text{ Hz}}{f} + \tan^{-1}\frac{45\text{ Hz}}{f} + \tan^{-1}\frac{68\text{ Hz}}{f} - \tan^{-1}\frac{f}{23\text{ kHz}} - \tan^{-1}\frac{f}{50\text{ kHz}}$$

$= \tan^{-1} 0.1 + \tan^{-1} 0.45 + \tan^{-1} 0.68 - \tan^{-1} 0.00435 - \tan^{-1} .002$
$= 5.71° + 24.23° + 34.22° - 0.249° - 0.115°$
= **63.8°** vs about 65° on the plot

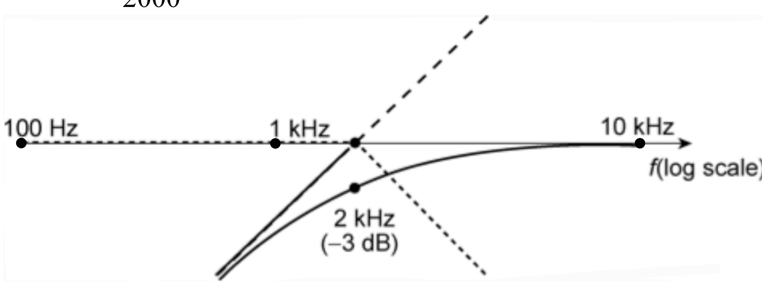
50. $f_{\text{low}} = f_{\text{high}} - BW = 36\text{ kHz} - 35.8\text{ kHz} = 0.2\text{ kHz} = 200\text{ Hz}$

$$A_\upsilon = \frac{-120}{\left(1 - j\frac{50}{f}\right)\left(1 - j\frac{200}{f}\right)\left(1 + j\frac{f}{36\text{ kHz}}\right)}$$



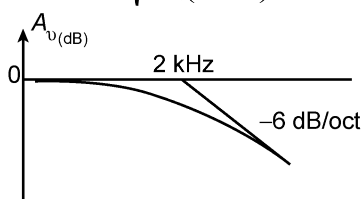
51. $$\mathbf{A}_\upsilon = \frac{0.05}{0.05 - j\frac{100}{f}} = \frac{1}{1 - j\frac{100}{0.05\,f}} = \frac{1}{1 - j\frac{2000}{f}} = \frac{+jf}{+jf + 2000}$$

$$= \frac{+j\frac{f}{2000}}{1 + j\frac{f}{2000}} \text{ and } f_1 = 2000\text{ Hz}$$

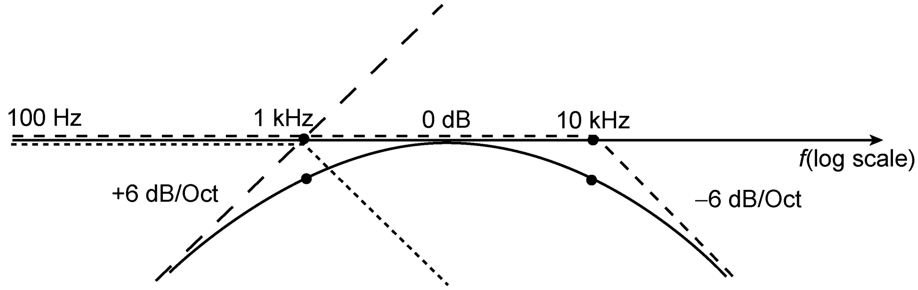


52. $$\mathbf{A}_\upsilon = \frac{200}{200 + j0.1f} = \frac{1}{1 + j\frac{0.1f}{200}} = \frac{1}{1 + j\frac{f}{2000}}$$

$$A_{\upsilon\text{dB}} = -20\log_{20}\frac{1}{\sqrt{1 + \left(\frac{f}{2000}\right)^2}}, \quad \frac{f}{2000} = 1 \text{ and } f = \mathbf{2\text{ kHz}}$$

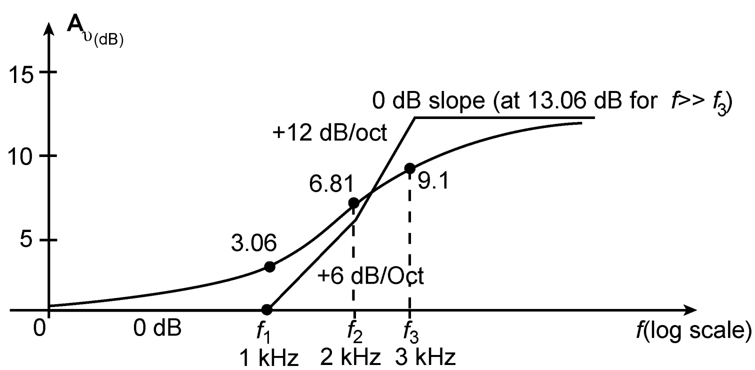


53. $\mathbf{A}_\upsilon = \dfrac{jf/1000}{(1+jf/1000)(1+jf/10{,}000)}$

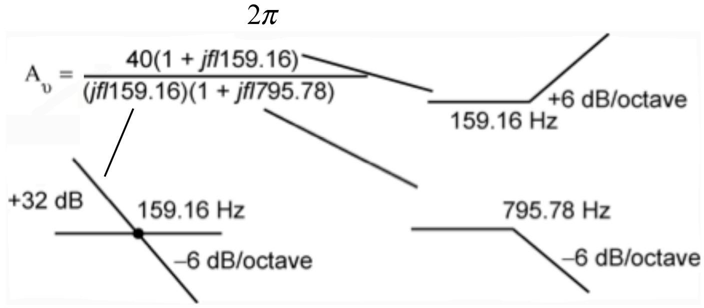


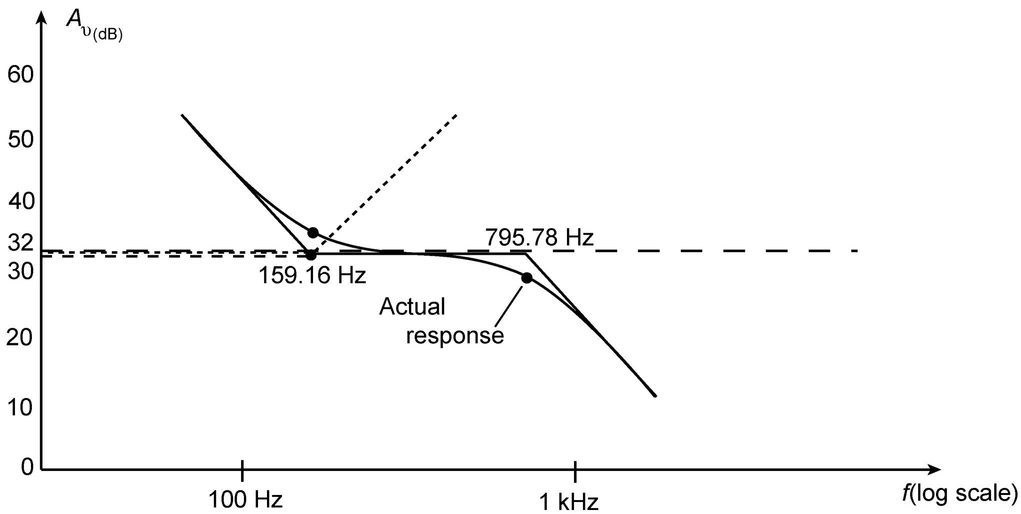
54. $\mathbf{A}_\upsilon = \dfrac{\left(1+j\dfrac{f}{1000}\right)\left(1+j\dfrac{f}{2000}\right)}{\left(1+j\dfrac{f}{3000}\right)^2}$

$$A_{\upsilon\text{dB}} = -20\log_{10}\sqrt{1+\left(\frac{f_1}{1000}\right)^2} + 20\log_{10}\sqrt{1+\left(\frac{f_2}{2000}\right)^2} + 40\log_{10}\frac{1}{\sqrt{1+\left(\frac{f_3}{3000}\right)^2}}$$



55. $\dfrac{j\omega}{1000} = j\dfrac{2\pi f}{1000} = j\dfrac{f}{\dfrac{1000}{2\pi}} = j\dfrac{f}{159.16\text{ Hz}}, \ \dfrac{j\omega}{5000} = j\dfrac{f}{795.78\text{ Hz}}$





56. a. Woofer − 400 Hz:

$X_L = 2\pi fL = 2\pi(400 \text{ Hz})(4.7 \text{ mH}) = 11.81\ \Omega$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(400\text{ Hz})(39\ \mu\text{F})} = 10.20\ \Omega$

$R \parallel X_C = 8\ \Omega\ \angle 0° \parallel 10.20\angle{-90°} = 6.3\ \Omega\ \angle{-38.11°}$

$\mathbf{V}_o = \dfrac{(R \parallel X_C)(\mathbf{V}_i)}{(R \parallel X_C) + jX_L} = \dfrac{(6.3\ \Omega \angle -38.11°)(\mathbf{V}_i)}{(6.3\ \Omega \angle -38.11°) + j\,11.81}$

$\mathbf{V}_o = 0.673\ \angle{-96.11°}\ \mathbf{V}_i$

and $A_\upsilon = \dfrac{V_o}{V_i}$ = **0.673** vs desired 0.707 (off by less than 5%)

Tweeter − 5 kHz:

$X_L = 2\pi fL = 2\pi(5 \text{ kHz})(0.39 \text{ mH}) = 12.25\ \Omega$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(5\text{ kHz})(2.7\ \mu\text{F})} = 11.79\ \Omega$

$R \parallel X_L = 8\ \Omega\ \angle 0° \parallel 12.25\ \Omega\ \angle 90° = 6.7\ \Omega\ \angle 33.15°$

$\mathbf{V}_o = \dfrac{(6.7\ \Omega \angle 33.15°)(\mathbf{V}_i)}{(6.7\ \Omega \angle 33.15°) - j\,11.79}$

$\mathbf{V}_o = 0.678\ \angle 88.54°\ \mathbf{V}_i$

and $A_\upsilon = \dfrac{V_o}{V_i}$ = **0.678** vs 0.707 (off by less than 5%)

b. Woofer − 3 kHz:

$X_L = 2\pi fL = 2\pi(3 \text{ kHz})(4.7 \text{ mH}) = 88.59\ \Omega$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(3\text{ kHz})(39\ \mu\text{F})} = 1.36\ \Omega$

$R \parallel X_C = 8\ \Omega\ \angle 0° \parallel 1.36\ \Omega\ \angle{-90°} = 1.341\ \Omega\ \angle{-80.35°}$

$\mathbf{V}_o = \dfrac{(R \parallel X_C)(\mathbf{V}_i)}{(R \parallel X_C) + jX_L} = \dfrac{(1.341\ \Omega \angle -80.35°)(\mathbf{V}_i)}{(1.341\ \Omega \angle -80.35°) + j\,88.59}$

$\mathbf{V}_o = 0.015\ \angle{-170.2°}\ \mathbf{V}_i$

and $A_v = \frac{V_o}{V_i}$ = **0.015** vs desired 0 (excellent)

Tweeter − 3 kHz:

$X_L = 2\pi fL = 2\pi(3\text{ kHz})(0.39\text{ mH}) = 7.35\ \Omega$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(3\text{ kHz})(2.7\ \mu\text{F})} = 19.65\ \Omega$$

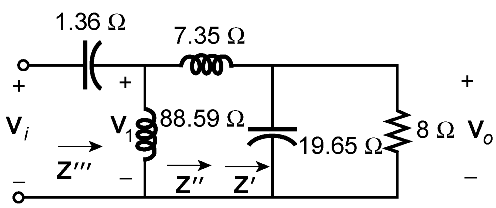
$R \parallel X_L = 8\ \Omega \angle 0° \parallel 7.35\ \Omega \angle 90° = 5.42\ \Omega \angle 47.42°$

$$\mathbf{V}_o = \frac{(R \parallel X_L)(\mathbf{V}_i)}{(R \parallel X_L) + jX_C} = \frac{(5.42\ \Omega \angle 47.42°)(\mathbf{V}_i)}{(5.42\ \Omega \angle 47.42°) - j19.65}$$

$\mathbf{V}_o = 0.337 \angle 124.24°\ \mathbf{V}_i$

and $A_v = \frac{V_o}{V_i}$ = **0.337** (acceptable since relatively close to cut frequency for tweeter)

c. Mid-range speaker − 3 kHz:



$\mathbf{Z}' = 7.41\ \Omega \angle -22.15°$

$\mathbf{Z}'' = 8.24\ \Omega \angle 33.58°$

$\mathbf{Z}''' = 7.816\ \Omega \angle 37.79°$

$$\mathbf{V}_1 = \frac{\mathbf{Z}'''\mathbf{V}_i}{\mathbf{Z}''' - jX_C} = \frac{(7.816\ \Omega \angle 37.79°)\mathbf{V}_i}{7.816\ \Omega \angle 37.79° - j1.36} = 1.11 \angle 8.83°\ \mathbf{V}_i$$

$$\mathbf{V}_o = \frac{\mathbf{Z}'\mathbf{V}_1}{\mathbf{Z}' + jX_L} = \frac{(7.41\ \Omega \angle -22.15°)\mathbf{V}_i}{7.41\ \Omega \angle -22.15° + j7.35} = 0.998 \angle -46.9°\ \mathbf{V}_i$$

and $A_v = \frac{V_o}{V_i}$ = **0.998** (excellent)