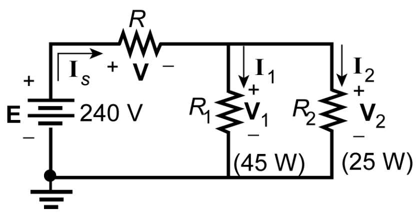
Chapter 20

1. a. $P_T = 60\text{ W} + 45\text{ W} + 25\text{ W} = \mathbf{130\ W}$

   b. $Q_T = \mathbf{0\ VARS}$, $S_T = P_T = \mathbf{130\ VA}$

   c. $S_T = EI_s$, $I_s = \dfrac{S_T}{E} = \dfrac{130\text{ VA}}{240\text{ V}} = \mathbf{0.542\ A}$

   d. 

$$P = I_s^2 R,\ R = \frac{P}{I_s^2} = \frac{60\text{ W}}{(0.542\text{ A})^2} = 204.2\ \Omega$$
$$V = I_sR = (0.542\text{ A})(204.2\ \Omega) = 110.68\text{ V}$$
$$V_1 = V_2 = E - V = 240\text{ V} - 110.68\text{ V} = 129.32\text{ V}$$
$$P_1 = \frac{V_1^2}{R_1},\ R_1 = \frac{V_1^2}{P_1} = \frac{(129.32\text{ V})^2}{45\text{ W}} = \mathbf{371.6\ \Omega}$$
$$P_2 = \frac{V_2^2}{R_2},\ R_2 = \frac{V_2^2}{P_2} = \frac{(129.32\text{ V})^2}{25\text{ W}} = \mathbf{668.9\ \Omega}$$

   e. $I_1 = \dfrac{V_1}{R_1} = \dfrac{129.32\text{ V}}{371.6\ \Omega} = \mathbf{0.348\ A}$, $I_2 = \dfrac{V_2}{R_2} = \dfrac{129.32\text{ V}}{668.9\ \Omega} = \mathbf{0.193\ A}$

2. a. $\mathbf{Z}_T = 3\ \Omega - j5\ \Omega + j9\ \Omega = 3\ \Omega + j4\ \Omega = 5\ \Omega\ \angle 53.13^\circ$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\text{ V}\ \angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ} = 10\text{ A}\ \angle -53.13^\circ$$

   R: $P = I^2R = (10\text{ A})^2\ 3\ \Omega = \mathbf{300\ W}$
   L: $P = \mathbf{0\ W}$
   C: $P = \mathbf{0\ W}$

   b. R: $Q = \mathbf{0\ VAR}$
   C: $Q_C = I^2X_C = (10\text{ A})^2\ 5\ \Omega = \mathbf{500\ VAR}$
   L: $Q_L = I^2X_L = (10\text{ A})^2\ 9\ \Omega = \mathbf{900\ VAR}$

   c. R: $S = \mathbf{300\ VA}$
   C: $S = \mathbf{500\ VA}$
   L: $S = \mathbf{900\ VA}$

   d. $P_T = 300\text{ W}$
   $Q_T = Q_L - Q_C = \mathbf{400\ VAR(\mathit{L})}$
   $S_T = \sqrt{P_T^2 + Q_T^2} = EI = (50\text{ V})(10\text{ A}) = \mathbf{500\ VA}$
   $F_p = \dfrac{P_T}{S_T} = \dfrac{300\text{ W}}{500\text{ VA}} = \mathbf{0.6\ lagging}$

   e. –

f. $W_R = \dfrac{VI}{f_1}$: $W_R = 2\left[\dfrac{VI}{f_2}\right] = 2\left[\dfrac{VI}{2f_1}\right] = \dfrac{VI}{f_1}$

$V = IR = (10\text{ A})(3\ \Omega) = 30\text{ V}$

$W_R = \dfrac{(30\text{ V})(10\text{ A})}{60\text{ Hz}} = \mathbf{5\ J}$

g. $V_C = IX_C = (10\text{ A})(5\ \Omega) = 50\text{ V}$

$W_C = \dfrac{VI}{\omega_1} = \dfrac{(50\text{ V})(10\text{ A})}{(2\pi)(60\text{ Hz})} = \mathbf{1.33\ J}$

$V_L = IX_L = (10\text{ A})(9\ \Omega) = 90\text{ V}$

$W_L = \dfrac{VI}{\omega_1} = \dfrac{(90\text{ V})(10\text{ A})}{376.8} = \mathbf{2.39\ J}$

3. a. R_1:

$\mathbf{I}_{R_1} = \dfrac{\mathbf{E}}{\mathbf{R}_1} = \dfrac{120\text{ V}\angle 0°}{2\text{ k}\Omega\angle 0°} = 60\text{ mA }\angle 0°$

$P_{R_1} = I^2R = (60\text{ mA})^2 2\text{ k}\Omega = \mathbf{7.2\ W}$

$Q_T = \mathbf{0\ VAR}$

$S_{R_1} = P_{R_1} = \mathbf{7.2\ VA}$

C:

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(5\text{ kHz})(0.02\ \mu\text{F})} = 3.98\text{ k}\Omega$

$P_T = \mathbf{0\ W}$

Q_T: $\mathbf{I}_C = \dfrac{\mathbf{E}}{\mathbf{X}_C} = \dfrac{120\text{ V}\angle 0°}{3.98\text{ k}\Omega\angle -90°} = 30.15\text{ mA }\angle 90°$

$Q_C = I^2X_C = (30.15\text{ mA})^2 3.98\text{ k}\Omega = \mathbf{3.62\ VAR}$

$S_T = Q_C = \mathbf{3.62\ VA}$

R_2–L:

$X_L = 2\pi fL = 2\pi(2\text{ kHz})(80\text{ mH}) = 1\text{ k}\Omega$

$\mathbf{Z}' = 200\ \Omega + j1\text{ k}\Omega = 1.02\text{ k}\Omega\angle 78.69°$

$\mathbf{I}_L = \dfrac{\mathbf{E}}{\mathbf{Z}'} = \dfrac{120\text{ V}\angle 0°}{1.02\text{ k}\Omega\angle 78.69°} = 117.65\text{ mA }\angle -78.69°$

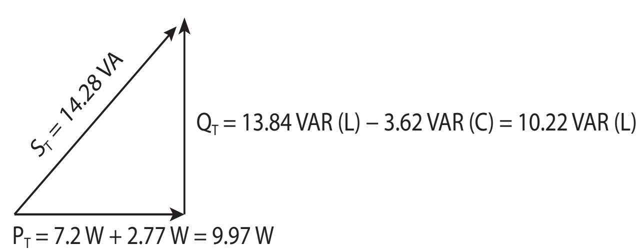
$Q_L = I^2X_L = (117.6\text{ mA})^2(1\text{ k}\Omega) = \mathbf{13.84\ VAR}$

$Q_T = Q_L = \mathbf{13.84\ VAR}$

$P_T = I^2R = (117.65\text{ mA})^2(200\ \Omega) = \mathbf{2.77\ W}$

$S_T = 2.77\text{ W} - j13.84\text{ VAR} \Rightarrow \mathbf{1.41\ VA}$

b.



$S_T = 9.97 + j10.22 = 14.28 \text{ VA} \angle 45.71°$

c. $\mathbf{Z'} = \mathbf{R}_1 \parallel \mathbf{X}_C = \dfrac{(2\text{ k}\Omega \angle 0°)(3.98\text{ k}\Omega \angle -90°)}{2\text{ k}\Omega - j3.98\text{ k}\Omega} = \dfrac{7.96\text{ k}\Omega \angle -90°}{4.45 \angle -63.32°}$

$= 1.79\text{ k}\Omega \angle -26.68°$

$\mathbf{Z}_T = \mathbf{Z'} \parallel (R_2 + jX_L) = \dfrac{(1.79\text{ k}\Omega \angle -26.68°)(1.02\text{ k}\Omega \angle 78.69°)}{(1.6\text{ k}\Omega - j0.8\text{ k}\Omega) + (0.2\text{ k}\Omega + j1\text{ k}\Omega)}$

$= \dfrac{1.83\text{ k}\Omega \angle 52.06°}{1.8\text{ k}\Omega + j0.2\text{ k}\Omega} = \dfrac{1.83\text{ k}\Omega \angle 52.06°}{1.81 \angle 6.34°}$

$= 1.01\text{ k}\Omega \angle 45.72°$

$F_P = \dfrac{P_T}{S_T} = \dfrac{9.97\text{ W}}{14.28\text{ VA}} = \mathbf{0.698\ lagging}$

d. $\mathbf{I}_s = \dfrac{\mathbf{E}}{\mathbf{Z}_T} = \dfrac{120\text{ V} \angle 0°}{1.01\text{ k}\Omega \angle 45.72°} = \mathbf{118.81\ mA \angle -45.72°}$

4. a. $P_T = 0 + 100\text{ W} + 300\text{ W} = \mathbf{400\ W}$

$Q_T = 200\text{ VAR}(L) - 600\text{ VAR}(C) + 0 = \mathbf{-400\ VAR}(\boldsymbol{C})$

$S_T = \sqrt{P_T^2 + Q_T^2} = \mathbf{565.69\ VA}$

$F_p = \dfrac{P_T}{S_T} = \dfrac{400\text{ W}}{565.69\text{ VA}} = \mathbf{0.707\ (leading)}$

b. –

c. $P_T = EI_s \cos\theta_T$

$400\text{ W} = (100\text{ V})I_s(0.7071)$

$I_s = \dfrac{400\text{ W}}{70.71\text{ V}} = 5.66\text{ A}$

$\mathbf{I}_s = 5.66\text{ A} \angle 135°$

5. a. $P_T = 600\text{ W} + 500\text{ W} + 100\text{ W} = \mathbf{1200\ W}$

$Q_T = 1200\text{ VAR}(L) + 600\text{ VAR}(L) - 1800(C) = \mathbf{0\ VAR}$

$S_T = P_T = \mathbf{1200\ VA}$

b. $F_p = \dfrac{P_T}{S_T} = \dfrac{1200\text{ W}}{1200\text{ VA}} = 1$

c. –

d. $I_s = \dfrac{S_T}{E} = \dfrac{1200\text{ VA}}{200\text{ V}} = 6\text{ A},\ 1 \Rightarrow 0°$

$\mathbf{I}_s = \mathbf{6\ A \angle 0°}$

6. a. $P_T = 200\text{ W} + 100\text{ W} + 0 + 50\text{ W} = \mathbf{350\text{ W}}$

$Q_T = 50\text{ VAR}(L) + 100\text{ VAR}(L) - 200\text{ VAR}(C) - 400\text{ VAR}(C) = \mathbf{-450\text{ VAR}(C)}$

$S_T = \sqrt{P_T^2 + Q_T^2} = \mathbf{570.09\text{ VA}}$

b. $F_p = \dfrac{P_T}{S_T} = \dfrac{350\text{ W}}{570.09\text{ VA}} = \mathbf{0.614\text{ (leading)}}$

c. –

d. $P_T = EI_s \cos\theta_T$

$350\text{ W} = (50\text{ V})I_s(0.614)$

$I_s = \dfrac{350\text{ W}}{30.7\text{ V}} = 11.4\text{ A}$

$\mathbf{I}_s = \mathbf{11.4\text{ A} \angle 52.12°}$

7. a. 200 W: Resistive:

$$P = I^2R \Rightarrow R = \frac{P}{I^2} = \frac{200\text{ W}}{(2\text{ A})^2} = \frac{200\text{ W}}{4\text{ A}^2} = \mathbf{50\ \Omega}$$

400 VAR(L): Inductive:

$$Q_L = I^2X_L \Rightarrow X_L = \frac{Q_L}{I^2} = \frac{200\text{ VAR}(L)}{4\text{ A}^2} = 100\ \Omega$$

$$X_L = 2\pi fL \Rightarrow L = \frac{X_L}{2\pi f} = \frac{100\ \Omega}{2\pi(5\text{ kHz})} = \mathbf{3.18\text{ mH}}$$

600 VAR(C): Capacitive:

$$Q_C = I^2X_C \Rightarrow X_C = \frac{Q_C}{I^2} = \frac{600\text{ VAR}(C)}{4\text{ A}^2} = 150\ \Omega$$

$$X_C = \frac{1}{2\pi fC} \Rightarrow C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi(5\text{ kHz})(150\ \Omega)} = \mathbf{212.2\text{ nF}}$$

b. $\mathbf{Z}_T = R + jX_L - jX_C = 50\ \Omega + j100\ \Omega - j150\ \Omega = 50\ \Omega - j50\ \Omega$

$= 70.71\ \Omega \angle -45°$

c. $\mathbf{V}_s = \mathbf{I}\mathbf{Z}_T = (2\text{ A} \angle 0°)(70.71\ \Omega \angle -45°)$

$= \mathbf{141.42\text{ V} \angle -45°}$

d. $F_p = \cos(45°) = \mathbf{0.707\text{ leading}}$

e. $\mathbf{V}_C = \mathbf{I}\mathbf{X}_C = (2\text{ A} \angle 0°)(150\ \Omega \angle -90°)$

$= \mathbf{300\text{ V} \angle -90°}$

8. a. $\mathbf{I}_R = \dfrac{60\text{ V} \angle 30°}{20\ \Omega \angle 0°} = 3\text{ A} \angle 30°$

$P = I^2R = (3\text{ A})^2\ 20\ \Omega = \mathbf{180\text{ W}}$

$Q_R = \mathbf{0\text{ VAR}}$

$S = P = \mathbf{180\text{ VA}}$

b. $\mathbf{I}_L = \dfrac{60\text{ V}\angle 30°}{10\,\Omega\angle 90°} = 6\text{ A}\angle -60°$

$P_L = \mathbf{0\ W}$

$Q_L = I^2X_L = (6\text{ A})^2\,10\,\Omega = \mathbf{360\ VAR(\mathit{L})}$

$S = Q_L = \mathbf{360\ VA}$

c. $P_T = 180\text{ W} + 400\text{ W} = \mathbf{580\ W}$

$Q_T = 600\text{ VAR}(L) + 360\text{ VAR}(L) = \mathbf{960\ VAR(\mathit{L})}$

$S_T = \sqrt{(580\text{ W})^2 + (960\text{ VAR})^2} = \mathbf{1121.61\ VA}$

$F_p = \dfrac{P_T}{S_T} = \dfrac{580\text{ W}}{1121.61\text{ VA}} = \mathbf{0.517\ (lagging)}\quad \theta = 58.87°$

d. $S_T = EI_s$

$I_s = \dfrac{S_T}{E} = \dfrac{1121.61\text{ VA}}{60\text{ V}} = 18.69\text{ A}$

$\theta_{I_s} = 30° - 58.87° = -28.87°$

$\mathbf{I}_s = \mathbf{18.69\ A\angle -28.87°}$

9. a. $R_3 + jX_L = 4\,\Omega + j4\,\Omega = 5.66\,\Omega\angle 45°$

$R_2 \parallel X_C\angle -90° = \dfrac{(2\,\Omega\angle 0°)(5\,\Omega\angle -90°)}{2\,\Omega - j5\,\Omega} = \dfrac{10\,\Omega\angle -90°}{5.39\angle -68.2°} = 1.86\,\Omega\angle -21.8°$

$\dfrac{(5.66\,\Omega\angle 45°)(1.86\,\Omega\angle -21.8°)}{(4\,\Omega - j4\,\Omega) + (1.73\,\Omega - j0.69\,\Omega)}$

$= \dfrac{10.53\,\Omega\angle 23.2°}{5.73 + j3.31} = \dfrac{10.53\,\Omega\angle 23.2°}{6.62\angle 30.01°} = 1.59\,\Omega\angle -6.81° = 1.58\,\Omega - j0.19\,\Omega$

$\mathbf{Z}_T = R_1 + 1.59\,\Omega\angle -6.81° = 2\,\Omega + 1.58\,\Omega - j0.19\,\Omega$

$= 3.58\,\Omega - j0.19\,\Omega = 3.59\,\Omega\angle -3.03°$

$\mathbf{I}_s = \dfrac{\mathbf{E}}{\mathbf{Z}_T} = \dfrac{20\text{ V}\angle 0°}{3.59\,\Omega\angle -3.03°} = \mathbf{5.57\ A\angle 3.03°}$

b. R_1: $P_{R_1} = I_s^2R_1 = (5.57\text{ A})^2 2\,\Omega = \mathbf{62.05\ W}$

$\mathbf{V}_{R_1} = \mathbf{I}_s\mathbf{R}_1 = (5.57\text{ A}\angle 3.03°)(2\,\Omega\angle 0°) = 11.14\text{ V}\angle 3.03°$

$\mathbf{V}_{R_2} = \mathbf{E} - \mathbf{V}_{R_1} = 20\text{ V}\angle 0° - 11.14\text{ V}\angle 3.03°$

$= 20\text{ V} - (11.12\text{ V} + j0.5\text{ V}) = 8.88\text{ V} - j0.59\text{ V}$

$= 8.89\text{ V}\angle -3.8°$

R_2: $P_{R_2} = \dfrac{V_{R_2}^2}{R_2} = \dfrac{(8.89\text{ V})^2}{2\,\Omega} = \mathbf{39.52\ W}$

$\mathbf{I}_{R_3} = \dfrac{\mathbf{V}_{R_2}}{R_3 + jX_L} = \dfrac{8.89\text{ V}\angle -3.8°}{4\,\Omega + j4\,\Omega} = \dfrac{8.89\text{ V}\angle -3.8°}{5.66\,\Omega\angle 45°} = 1.57\text{ A}\angle -48.8°$

R_3: $P_{R_3} = I_{R_3}^2 R_3 = (1.57\text{ A})^2 4\,\Omega = \mathbf{9.86\ W}$

c. X_C: $Q_{X_C} = \dfrac{V_{R_3}^{\,2}}{X_C} = \dfrac{(8.89\text{ V})^2}{5\ \Omega}$ = **15.81 VAR(*C*)**

X_L: $Q_L = I^2X_L = (1.57\text{ A})^2 4\ \Omega$ = **9.86 VAR(*L*)**

d. R_1: $S_T = P_{R_1}$ = **62.05 VA**

R_2: $S_T = P_{R_2}$ = **39.52 VA**

R_3: $S_T = P_{R_3}$ = **9.86 VA**

C: $S_T = Q_{X_C}$ = **15.81 VA**

L: $S_T = Q_L$ = **9.86 VA**

e. P_T: $P_{R_1} + P_{R_2} + P_{R_3}$ = 62.05 W + 39.52 W + 9.86 W

= **111.43 W**

Q_T: $Q_{X_C} - Q_{X_L}$ = 15.81 VAR(*C*) − 9.86 VAR(*L*)

= **5.95 VAR(*L*)**

$S_T = \sqrt{P_T^{\,2} + Q_T^{\,2}}$ = **111.59 VA**

$F_p = \dfrac{P_T}{S_T} = \dfrac{111.43\text{ W}}{111.59\text{ VA}}$ = **0.998 (leadering)**

f. −

10. a. $\mathbf{Z}_1 = 3\ \Omega + j4\ \Omega = 5\ \Omega \angle 53.13°$

$\mathbf{Z}_2 = 3\ \Omega - j4\ \Omega = 5\ \Omega \angle -53.13°$

$\mathbf{Z}_T = \mathbf{Z}_1 \parallel \mathbf{Z}_2 = \dfrac{(5\ \Omega \angle 53.13°)(5\ \Omega \angle -53.13°)}{(3\ \Omega + j4\ \Omega) + (3\ \Omega - j4\ \Omega)} = \dfrac{25\ \Omega \angle 0°}{6} = 4.17\ \Omega \angle 0°$

$\mathbf{I}_s = \dfrac{\mathbf{E}}{\mathbf{Z}_T} = \dfrac{50\text{ V} \angle 60°}{4.17\ \Omega \angle 0°}$ = **11.99 A ∠60°**

b. $\mathbf{I}_{R-L} = \dfrac{\mathbf{E}}{\mathbf{Z}_{R-L}} = \dfrac{50\text{ V} \angle 60°}{5\ \Omega \angle 53.13°} = 10\text{ A} \angle 16.87°$

$P_{3\Omega} = I^2R = (10\text{ A})^2 \cdot 3\ \Omega$ = **300 W**

$\mathbf{I}_{R-C} = \dfrac{\mathbf{E}}{\mathbf{Z}_{R-C}} = \dfrac{50\text{ V} \angle 60°}{5\ \Omega \angle -53.13°} = 10\text{ A} \angle 113.13°$

$P_{3\Omega} = I^2R = (10\text{ A})^2 \cdot 3\ \Omega$ = **300 W**

c. L: $\mathbf{I}_{R-L} = 10\text{ A} \angle 16.87°$

$Q_L = I^2X_L = (10\text{ A})^2 \cdot 4\ \Omega$ = **400 VAR(*L*)**

C: $\mathbf{I}_{R-C} = 10\text{ A} \angle 113.13°$

$Q_C = I^2X_C = (10\text{ A})^2 \cdot 4\ \Omega$ = **400 VAR(*C*)**

d. 3 Ω: $S_T = P_{3\Omega}$ = **300 VA** for each resistor

L: $S_T = Q_L$ = **400 VA**

C: $S_T = Q_C$ = **400 VA**

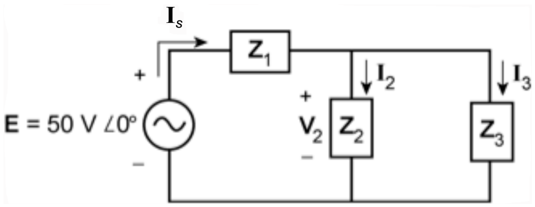
e. P_T = 300 W + 300 W = **600 W**

$Q_T = 400\text{ VAR}(L) + 400\text{ VAR}(C) = \mathbf{0\ VAR}$

$S_T = P_T = \mathbf{600\ VA}$

$$F_p = \frac{P_T}{S_T} = \frac{600\text{ W}}{600\text{ VA}} = \mathbf{1}$$

11. a–c.



$X_L = \omega L = (400\text{ rad/s})(0.1\text{ H}) = 40\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{(400\text{ rad/s})(100\ \mu\text{F})} = 25\ \Omega$$

$\mathbf{Z}_1 = 40\ \Omega\ \angle 90°$, $\mathbf{Z}_2 = 25\ \Omega\ \angle -90°$

$\mathbf{Z}_3 = 30\ \Omega\ \angle 0°$

$$\begin{aligned}\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 &= +j40\ \Omega + (25\ \Omega\ \angle -90°) \parallel (30\ \Omega\ \angle 0°)\\ &= +j40\ \Omega + 19.21\ \Omega\ \angle -50.19°\\ &= +j40\ \Omega + 12.3\ \Omega - j14.76\ \Omega\\ &= 12.3\ \Omega + j25.24\ \Omega\\ &= 28.08\ \Omega\ \angle 64.02°\end{aligned}$$

$$\mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\text{ V}\ \angle 0°}{28.08\ \Omega\ \angle 64.02°} = 1.78\text{ A}\ \angle -64.02°$$

$$\begin{aligned}\mathbf{V}_2 = \mathbf{I}_s(\mathbf{Z}_2 \parallel \mathbf{Z}_3) &= (1.78\text{ A}\ \angle -64.02°)(19.21\ \Omega\ \angle -50.19°)\\ &= 34.19\text{ V}\ \angle -114.21°\end{aligned}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{\mathbf{Z}_2} = \frac{34.19\text{ V}\ \angle -114.21°}{25\ \Omega\ \angle -90°} = 1.37\text{ A}\ \angle -24.21°$$

$$\mathbf{I}_3 = \frac{\mathbf{V}_2}{\mathbf{Z}_3} = \frac{34.19\text{ V}\ \angle -114.21°}{30\ \Omega\ \angle 0°} = 1.14\text{ A}\ \angle -114.21°$$

$\mathbf{Z}_1$: $P = \mathbf{0\ W}$, $Q_L = I_s^2 X_L = (1.78\text{ A})^2\ 40\ \Omega = \mathbf{126.74\ VAR(\mathit{L})}$, $S = \mathbf{126.74\ VA}$

$\mathbf{Z}_2$: $P = \mathbf{0\ W}$, $Q_C = I_2^2 X_C = (1.37\text{ A})^2\ 25\ \Omega = \mathbf{46.92\ VAR(\mathit{C})}$, $S = \mathbf{46.92\ VA}$

$\mathbf{Z}_3$: $P = I_3^2 R = (1.14\text{ A})^2\ 30\ \Omega = \mathbf{38.99\ W}$, $Q_R = \mathbf{0\ VAR}$, $S = \mathbf{38.99\ VA}$

d. $P_T = 0 + 0 + 38.99\text{ W} = \mathbf{38.99\ W}$

$Q_T = +126.74\text{ VAR}(L) - 46.92\text{ VAR}(C) + 0 = \mathbf{79.82\ VAR(\mathit{L})}$

$S_T = \sqrt{P_T^2 + Q_T^2} = \mathbf{88.83\ VA}$

$$F_p = \frac{P_T}{S_T} = \frac{38.99\text{ W}}{88.83\text{ VA}} = \mathbf{0.439\ (lagging)}$$

e. –

f. $$W_R = \frac{V_R I_R}{2f_1} = \frac{V_2 I_3}{2f_1} = \frac{(34.19\text{ V})(1.14\text{ A})}{2(63.69\text{ Hz})} = \mathbf{0.31\ J}$$

$$f_1 = \frac{\omega_1}{2\pi} = \frac{400\text{ rad/s}}{6.28} = 63.69\text{ Hz}$$

g. $W_L = \dfrac{V_L I_L}{\omega_1} = \dfrac{(I_s X_L) I_s}{\omega_1} = \dfrac{I_s^2 X_L}{\omega_1} = \dfrac{(1.78\text{ A})^2\ 40\ \Omega}{400\text{ rad/s}}$ = **0.32 J**

$W_C = \dfrac{V_C I_C}{\omega_1} = \dfrac{V_2 I_2}{\omega_1} = \dfrac{(34.19\text{ V})(1.37\text{ A})}{400\text{ rad/s}}$ = **0.12 J**

12. a. $I_s = \dfrac{S_T}{E} = \dfrac{10{,}000\text{ VA}}{200\text{ V}}$ = 50 A

0.5 ⇒ 60° leading

∴ $\mathbf{I}_s$ leads **E** by 60°

$\mathbf{Z}_T = \dfrac{\mathbf{E}}{\mathbf{I}_s} = \dfrac{200\text{ V}\angle 0°}{50\text{ A}\angle 60°}$ = 4 Ω ∠–60° = 2 Ω – j3.464 Ω = $R - jX_C$

b. $F_p = \dfrac{P_T}{S_T} \Rightarrow P_T = F_p S_T$ = (0.5)(10,000 VA) = **5000 W**

13. a. $I = \dfrac{S_T}{E} = \dfrac{5000\text{ VA}}{120\text{ V}}$ = 41.67 A

F_p = 0.8 ⇒ 36.87° (lagging)

E = 120 V ∠0°, **I** = 41.67 A ∠–36.87°

$\mathbf{Z} = \dfrac{\mathbf{E}}{\mathbf{I}} = \dfrac{120\text{ V}\angle 0°}{41.67\text{ A}\angle -36.87°}$ = 2.88 Ω ∠36.87° = **2.30 Ω + *j*1.73 Ω** = $R + jX_L$

b. $P = S\cos\theta$ = (5000 VA)(0.8) = **4000 W**

14. a. P_T = 0 + 300 W = **300 W**

Q_T = 600 VAR(C) + 200(L) = **400 VAR(*C*)**

$S_T = \sqrt{P_T^2 + Q_T^2}$ = **500 VA**

$F_p = \dfrac{P_T}{S_T} = \dfrac{300\text{ W}}{500\text{ VA}}$ = **0.6 (leading)**

b. $I_s = \dfrac{S_T}{E} = \dfrac{500\text{ VA}}{30\text{ V}}$ = 16.67 A

F_p = 0.6 ⇒ 53.13°

$\mathbf{I}_s$ = **16.67 A ∠53.13°**

c. –

d. Load: 600 VAR(C), 0 W

$R = 0$, $L = 0$, $Q_C = I^2 X_C \Rightarrow X_C = \dfrac{Q_C}{I^2} = \dfrac{600\text{ VAR}}{(16.67\text{ A})^2}$ = **2.159 Ω**

Load: 200 VAR(L), 300 W

$C = 0$, $R = P/I^2$ = 300 W/(16.67 A)^2 = **1.079 Ω**

$X_L = \dfrac{Q_L}{I^2} = \dfrac{200\text{ VAR}}{(16.67\text{ A})^2}$ = **0.7197 Ω**

$\mathbf{Z}_T$ = –j2.159 Ω + 1.0796 Ω + j0.7197 Ω

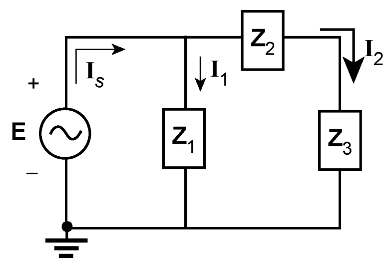
= **1.08 Ω – *j*1.44 Ω**

15. a. $P_T = 0 + 300\text{ W} + 600\text{ W} = \mathbf{900\ W}$
$Q_T = 500\text{ VAR}(C) + 0 + 500\text{ VAR}(L) = \mathbf{0\ VAR}$
$S_T = P_T = \mathbf{900\ VA}$
$$F_p = \frac{P_T}{S_T} = \mathbf{1}$$

b. $$I_s = \frac{S_T}{E} = \frac{900\text{ VA}}{100\text{ V}} = 9\text{ A},\ \mathbf{I}_s = \mathbf{9\ A\ \angle 0°}$$

c. –

d.

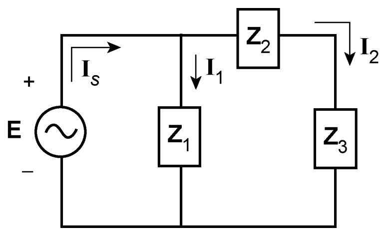


$\mathbf{Z}_1$: $$Q_C = \frac{V^2}{X_C} \Rightarrow X_C = \frac{V^2}{Q_C} = \frac{10^4}{500} = \mathbf{20\ \Omega}$$
$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{100\text{ V}\angle 0°}{20\ \Omega \angle -90°} = 5\text{A} \angle 90°$$
$$\mathbf{I}_2 = \mathbf{I}_s - \mathbf{I}_1 = 9\text{ A} - j5\text{ A} = 10.296\text{ A} \angle -29.05°$$
$\mathbf{Z}_2$: $$R = \frac{P}{I^2} = \frac{300\text{ W}}{(10.296\text{ A})^2} = \frac{300}{106} = \mathbf{2.83\ \Omega}$$
$X_{L,C} = \mathbf{0\ \Omega}$
$\mathbf{Z}_3$: $$R = \frac{P}{I_2^2} = \frac{600\text{ W}}{(10.296\text{ A})^2} = \mathbf{5.66\ \Omega}$$
$$X_L = \frac{Q}{I_2^2} = \frac{500}{(10.296\text{ A})^2} = \mathbf{4.72\ \Omega},\ X_C = \mathbf{0\ \Omega}$$

16. a. $P_T = 200\text{ W} + 30\text{ W} + 0 = \mathbf{230\ W}$
$Q_T = 0 + 40\text{ VAR}(L) + 100\text{ VAR}(L) = \mathbf{140\ VAR(\mathit{L})}$
$$S_T = \sqrt{P_T^2 + Q_T^2} = \mathbf{269.26\ VA}$$
$$F_p = \frac{P_T}{S_T} = \frac{230\text{ W}}{269.26\text{ VA}} = \mathbf{0.854\ (lagging)} \Rightarrow 31.35°$$

b. $$I_s = \frac{S_T}{E} = \frac{269.26\text{ VA}}{100\text{ V}} = 2.6926\text{ A}$$
$\mathbf{I}_s = \mathbf{2.69\ A\ \angle -31.35°}$

c.



$\mathbf{Z}_1$: $R = \dfrac{V^2}{P} = \dfrac{10^4}{200} = 50\ \Omega$

$X_L, X_C = 0\ \Omega$

$\mathbf{I}_1 = \dfrac{100\text{ V}\angle 0°}{50\ \Omega\angle 0°} = 2\text{ A}\angle 0°$

$\mathbf{I}_2 = \mathbf{I}_s - \mathbf{I}_1$

$= 2.6926\text{ A}\angle{-31.35°} - 2\text{ A}\angle 0°$

$= 2.299\text{ A} - j1.40\text{ A} - 2.0\text{ A}$

$= 0.299\text{ A} - j1.40\text{ A}$

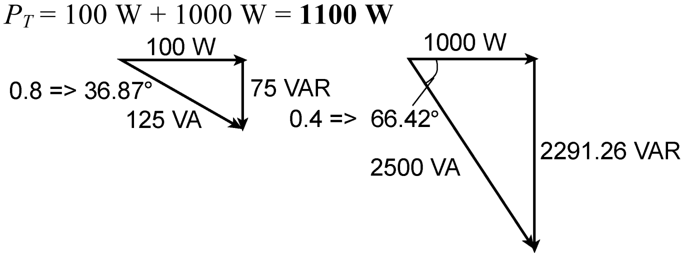
$= 1.432\text{ A}\angle{-77.94°}$

$\mathbf{Z}_2$: $R = \dfrac{P}{I_2^2} = \dfrac{30\text{ W}}{(1.432\text{ A})^2} = \mathbf{14.63\ \Omega}$, $X_L = \dfrac{Q}{I_2^2} = \dfrac{40\text{ VAR}}{(1.432\text{ A})^2} = \mathbf{19.50\ \Omega}$

$X_C = \mathbf{0\ \Omega}$

$\mathbf{Z}_3$: $X_L = \dfrac{Q}{I_2^2} = \dfrac{100\text{ VAR}}{(1.432\text{ A})^2} = \mathbf{48.76\ \Omega}$, $R = \mathbf{0\ \Omega}$, $X_C = \mathbf{0\ \Omega}$

17. a. $P_T = 100\text{ W} + 1000\text{ W} = \mathbf{1100\ W}$



$Q_T = 75\text{ VAR}(C) + 2291.26\text{ VAR}(C) = \mathbf{2366.26\ VAR}(\boldsymbol{C})$

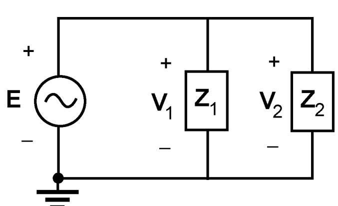
$S_T = \sqrt{P_T^2 + Q_T^2} = \mathbf{2609.44\ VA}$

$F_p = \dfrac{P_T}{S_T} = \dfrac{1100\text{ W}}{2609.44\text{ VA}} = \mathbf{0.422\ (leading)} \Rightarrow 65.04°$

b. $S_T = EI \Rightarrow E = \dfrac{S_T}{I} = \dfrac{2609.44\text{ VA}}{5\text{ A}} = 521.89\text{ V}$

$\mathbf{E} = \mathbf{521.89\ V}\angle\mathbf{-65.07°}$

c.



$I_{Z_1} = \dfrac{S}{V_1} = \dfrac{S}{E} = \dfrac{125\text{ VA}}{521.89\text{ V}} = 0.2395\text{ A}$

$$I_{Z_2} = \frac{S}{V_2} = \frac{S}{E} = \frac{2500\text{ VA}}{521.89\text{ V}} = 4.79\text{ A}$$

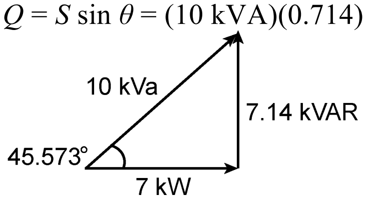
$\mathbf{Z}_1$: $R = \dfrac{P}{I_{Z_1}^2} = \dfrac{100\text{ W}}{(0.2395)^2} = \mathbf{1743.38\ \Omega}$

$$Q = I_{Z_1}^2 X_C \Rightarrow X_C = \frac{Q}{I_{Z_1}^2} = \frac{75\text{ VAR}}{(0.2395\text{ A})^2} = \mathbf{1307.53\ \Omega}$$

$\mathbf{Z}_2$: $R = \dfrac{P}{I_{Z_2}^{\ 2}} = \dfrac{1000\text{ W}}{(4.790\text{ A})^2} = \mathbf{43.59\ \Omega}$

$$X_C = \frac{Q}{I_{Z_2}^{\ 2}} = \frac{2291.26\text{ VAR}}{(4.790\text{ A})^2} = \mathbf{99.88\ \Omega}$$

18. a. $0.7 \Rightarrow 45.573°$
$P = S\cos\theta = (10\text{ kVA})(0.7) = 7\text{ kW}$
$Q = S\sin\theta = (10\text{ kVA})(0.714) = 7.14\text{ kVAR}(L)$



b. $Q_C = 7.14\text{ kVAR} = \dfrac{V^2}{X_C}$

$$X_C = \frac{V^2}{Q_C} = \frac{(208\text{ V})^2}{7.14\text{ kVAR}} = 6.059\ \Omega$$

$$X_C = \frac{1}{2\pi fC} \Rightarrow C = \frac{1}{2\pi fX_C} = \frac{1}{(2\pi)(60\text{ Hz})(6.059\ \Omega)} = \mathbf{438\ \mu F}$$

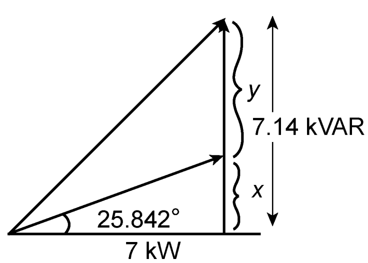
c. Uncompensated:

$$I_s = \frac{S_T}{E} = \frac{10{,}000\text{ VA}}{208\text{ V}} = \mathbf{48.08\ A}$$

Compensated:

$$I_s = \frac{S_T}{E} = \frac{P_T}{E} = \frac{7{,}000\text{ W}}{208\text{ V}} = \mathbf{33.65\ A}$$

d.



$\cos\theta = 0.9$
$\theta = \cos^{-1}0.9 = 25.842°$

$\tan\theta = \dfrac{x}{7\text{ kW}}$

$x = (7\text{ kW})(\tan 25.842°)$
$= (7\text{ kW})(0.484)$
$= 3.39\text{ kVAR}$
$y = (7.14 - 3.39)\text{ kVAR}$
$= 3.75\text{ kVAR}$

$Q_C = 3.75 \text{ kVAR} = \dfrac{V^2}{X_C}$

$X_C = \dfrac{V^2}{Q_C} = \dfrac{(208\text{ V})^2}{3.75\text{ kVAR}} = 11.537\ \Omega$

$C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{(2\pi)(60\text{ Hz})(11.537\ \Omega)} = \mathbf{230\ \mu F}$

Uncompensated:

$I_s = \mathbf{48.08\ A}$

Compensated:

$S_T = \sqrt{(7\text{ kW})^2 + (3.39\text{ kVAR})^2} = 7.778\text{ kVA}$

$I_s = \dfrac{S_T}{E} = \dfrac{7.778\text{ kVA}}{208\text{ V}} = \mathbf{37.39\ A}$

$\Delta I_s = 48.08\text{ A} - 37.39\text{ A} = \mathbf{10.69\ A}$

19. a. $P_T = 5\text{ kW},\ Q_T = 6\text{ kVAR}(L)$

$S_T = \sqrt{P_T^2 + Q_T^2} = \mathbf{7.81\ kVA}$

b. $F_p = \dfrac{P_T}{S_T} = \dfrac{5\text{ kW}}{7.81\text{ kVA}} = \mathbf{0.640\ (lagging)}$

c. $I_s = \dfrac{S_T}{E} = \dfrac{7{,}810\text{ VA}}{120\text{ V}} = \mathbf{65.08\ A}$

d. $X_C = \dfrac{1}{2\pi f C},\ Q_C = I^2 X_C = \dfrac{E^2}{X_C} = \dfrac{(120\text{ V})^2}{X_C}$

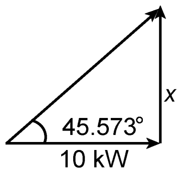
and $X_C = \dfrac{(120\text{ V})^2}{Q_C} = \dfrac{14{,}400}{6000} = 2.4\ \Omega$

$C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{(2\pi)(60\text{ Hz})(2.4\ \Omega)} = \mathbf{1105\ \mu F}$

e. $S_T = EI_s = P_T$

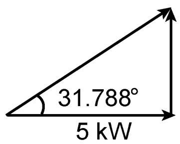
$\therefore\ I_s = \dfrac{P_T}{E} = \dfrac{5000\text{ W}}{120\text{ V}} = \mathbf{41.67\ A}$

20. a. Load 1: P = 20,000 W, Q = 0 VAR
Load 2: $\theta = \cos^{-1}0.7 = 45.573°$



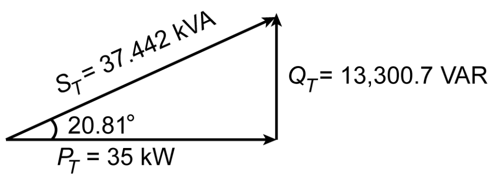
$$\tan\theta = \frac{x}{10\text{ kW}}$$
x = (10 kW)tan 45.573°
= (10 kW)(1.02)
= **10,202 VAR(*L*)**

Load 3: $\theta = \cos^{-1}0.85 = 31.788°$



$$\tan\theta = \frac{x}{5\text{ kW}}$$
x = (5 kW)tan 31.788°
= (5 kW)(0.62)
= 3098.7 VAR(*L*)

P_T = 20,000 W + 10,000 W + 5,000 W = **35 kW**
Q_T = 0 + 10,202 VAR + 3098.7 VAR = **13,300.7 VAR(*L*)**
$S_T = \sqrt{P_T^2 + Q_T^2}$ = **37.442 kVA**



b. $Q_C = Q_L$ = 13,300.7 VAR
$$X_C = \frac{E^2}{Q_C} = \frac{(10^3\text{ V})^2}{13{,}300.7\text{ VAR}} = 75.184\ \Omega$$
$$C = \frac{1}{2\pi f X_C} = \frac{1}{(2\pi)(60\text{ Hz})(75.184\ \Omega)} = \mathbf{35.28\ \mu F}$$

c. Uncompensated:
$$I_s = \frac{S_T}{E} = \frac{37.442\text{ kVA}}{1\text{ kV}} = \mathbf{37.44\ A}$$

Compensated:
$S_T = P_T$ = 35 kW
$$I_s = \frac{S_T}{E} = \frac{35\text{ kW}}{1\text{ kV}} = \mathbf{35\ A}$$
ΔI_s = 37.44 A - 35 A = **2.44 A**

21. a. $\mathbf{Z}_T = R_1 + R_2 + R_3 + jX_L - jX_C$
= 2 Ω + 3 Ω + 1 Ω + j3 Ω – j12 Ω = 6 Ω – j9 Ω = 10.82 Ω ∠–56.31°
$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\text{ V}\angle 0°}{10.82\ \Omega\angle -56.31°} = 4.62\text{ A}\angle 56.31°$$
$P = VI\cos\theta$ = (50 V)(4.62 A) cos 56.31° = **128.14 W**

b. a-b: $P = I^2R = (4.62\text{ A})^2\ 2\ \Omega =$ **42.69 W**
b-c: $P = I^2R = (4.62\text{ A})^2\ 3\ \Omega =$ **64.03 W**
a-c: 42.69 W + 64.03 W = **106.72 W**
a-d: **106.72 W**
c-d: **0 W**
d-e: **0 W**
f-e: $P = I^2R = (4.62\text{ A})^2\ 1\ \Omega =$ **21.34 W**

22. a. $S_T = 660\text{ VA} = EI_s$

$$I_s = \frac{660\text{ VA}}{120\text{ V}} = 5.5\text{ A}$$

$$\theta = \cos^{-1}0.6 = 53.13°$$

$$\therefore \mathbf{E} = 120\text{ V}\angle 0°,\ \mathbf{I}_s = 5.5\text{ A}\angle -53.13°$$

$P = EI\cos\theta = (120\text{ V})(5.5\text{ A})(0.6) =$ **396 W**

Wattmeter = **396 W**, Ammeter = **5.5 A**, Voltmeter = **120 V**

b. $$\mathbf{Z}_T = \frac{\mathbf{E}}{\mathbf{I}} = \frac{120\text{ V}\angle 0°}{5.5\text{ A}\angle -53.13°} = \mathbf{21.82\ \Omega\angle 53.13° = 13.09\ \Omega + \mathit{j}17.46\ \Omega} = R + jX_L$$

23. a. $$R = \frac{P}{I^2} = \frac{80\text{ W}}{(4\text{ A})^2} = \mathbf{5\ \Omega},\ \mathbf{Z}_T = \frac{E}{I} = \frac{200\text{ V}}{4\text{ A}} = 50\ \Omega$$

$$X_L = \sqrt{Z_T^{\,2} - R^2} = \sqrt{(50\ \Omega)^2 - (5\ \Omega)^2} = 49.75\ \Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{49.57\ \Omega}{(2\pi)(60\text{ Hz})} = \mathbf{132.03\text{ mH}}$$

b. $$R = \frac{P}{I^2} = \frac{90\text{ W}}{(3\text{ A})^2} = \mathbf{10\ \Omega}$$

c. $$R = \frac{P}{I^2} = \frac{60\text{ W}}{(2\text{ A})^2} = \mathbf{15\ \Omega},\ Z_T = \frac{E}{I} = \frac{200\text{ V}}{2\text{ A}} = 100\ \Omega$$

$$X_L = \sqrt{Z_T^2 - R^2} = \sqrt{(100\ \Omega)^2 - (15\ \Omega)^2} = 98.87\ \Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{98.87\ \Omega}{376.8} = \mathbf{262.39\text{ mH}}$$

24. a. $X_L = 2\pi fL = (6.28)(50\text{ Hz})(0.08\text{ H}) = 25.12\ \Omega$

$$Z_T = \sqrt{R^2 + X_L^2} = \sqrt{(4\ \Omega)^2 + (25.12\ \Omega)^2} = 25.44\ \Omega$$

$$I = \frac{E}{Z_T} = \frac{60\text{ V}}{25.44\ \Omega} = 2.358\text{ A}$$

$P = I^2R = (2.358\text{ A})^2\ 4\ \Omega =$ **22.24 W**

b. $$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{30\text{ W}}{7\ \Omega}} = \mathbf{2.07\text{ A}}$$

$$Z_T = \frac{E}{I} = \frac{60\text{ V}}{2.07\text{ A}} = 28.99\ \Omega$$

$$X_L = \sqrt{(28.99\ \Omega)^2 - (7\ \Omega)^2} = 28.13\ \Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{28.13\ \Omega}{(2\pi)(50\ \text{Hz})} = \mathbf{89.54\ mH}$$

c. $P = I^2R = (1.7\ \text{A})^2\ 10\ \Omega = \mathbf{28.9\ W}$

$$Z_T = \frac{E}{I} = \frac{60\ \text{V}}{1.7\ \text{A}} = 35.29\ \Omega$$

$$X_L = \sqrt{(35.29\ \Omega)^2 - (10\ \Omega)^2} = 33.84\ \Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{38.84\ \Omega}{314} = \mathbf{107.77\ mH}$$