Chapter 18

1. –

2. $\mathbf{Z} = -j5\ \Omega + 2\ \Omega\ \angle 0° \parallel 5\ \Omega\ \angle 90° = -j5\ \Omega + 1.72\ \Omega + j0.69\ \Omega = \mathbf{4.64\ \Omega\ \angle -68.24°}$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{60\text{ V}\ \angle 30°}{4.64\ \Omega\ \angle -68.24°} = \mathbf{12.93\ A\ \angle 98.24°}$$

3. $\mathbf{Z} = 10\ \Omega\ \angle 0° \parallel 6\ \Omega\ \angle 90° = \mathbf{5.15\ \Omega\ \angle 59.04°}$

$\mathbf{E} = \mathbf{IZ} = (2\text{ A}\ \angle 120°)(5.15\ \Omega\ \angle 59.04°)$

$= \mathbf{10.30\ V\ \angle 179.04°}$

4. a. $\mathbf{I} = \dfrac{\mu \mathbf{V}}{R} = \dfrac{16\ \mathbf{V}}{4 \times 10^3} = \mathbf{4 \times 10^{-3}\ V}$

$\mathbf{Z = 4\ k\Omega\ \angle 0°}$

b. $\mathbf{V} = (h\mathbf{I})(R) = (50\ \mathbf{I})(50\text{ k}\Omega) = \mathbf{2.5 \times 10^6\ I}$

$\mathbf{Z = 50\ k\Omega\ \angle 0°}$

5. Clockwise mesh currents:

$\mathbf{E} - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2\mathbf{Z}_2 = 0$

$-\mathbf{I}_2\mathbf{Z}_2 + \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 = 0$

$[\mathbf{Z}_1 + \mathbf{Z}_2]\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 = \mathbf{E}_1$

$-\mathbf{Z}_2\mathbf{I}_1 + [\mathbf{Z}_2 + \mathbf{Z}_3]\mathbf{I}_2 = -\mathbf{E}_2$

$\mathbf{Z}_1 = R_1\ \angle 0° = 4\ \Omega\ \angle 0°$

$\mathbf{Z}_2 = X_L\ \angle 90° = 6\ \Omega\ \angle 90°$

$\mathbf{Z}_3 = X_C\ \angle -90° = 8\ \Omega\ \angle -90°$

$\mathbf{E}_1 = 10\text{ V}\ \angle 0°$, $\mathbf{E}_2 = 40\text{ V}\ \angle 60°$

$$\mathbf{I}_{R_1} = \mathbf{I}_1 = \frac{\begin{vmatrix} \mathbf{E}_1 & -\mathbf{Z}_2 \\ -\mathbf{E}_2 & [\mathbf{Z}_2 + \mathbf{Z}_3] \end{vmatrix}}{\begin{vmatrix} [\mathbf{Z}_1 + \mathbf{Z}_2] & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & [\mathbf{Z}_2 + \mathbf{Z}_3] \end{vmatrix}} = \frac{[\mathbf{Z}_2 + \mathbf{Z}_3]\mathbf{E}_1 - \mathbf{Z}_2\mathbf{E}_2}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3} = \mathbf{5.15\ A\ \angle -24.5°}$$

6. By interchanging the right two branches, the general configuration of Problem 5 will result and

$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1 - \mathbf{E}_2$

$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 = \mathbf{E}_2$

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and $\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1 - \mathbf{E}_2$

$-\mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) = \mathbf{E}_2$

$\mathbf{I}_{50\Omega} = \mathbf{I}_1 = \mathbf{145.45\ mA\ \angle 187.59°}$

$\mathbf{Z}_1 = R_1 = 50\ \Omega\ \angle 0°$

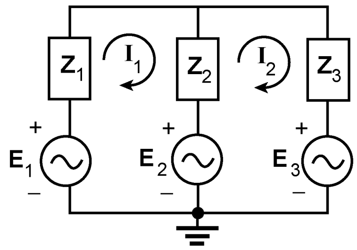
$\mathbf{Z}_2 = X_C\ \angle -90° = 40\ \Omega\ \angle -90°$

$\mathbf{Z}_3 = X_C\ \angle -90° = 60\ \Omega\ \angle -90°$

$\mathbf{E}_1 = 5\text{ V}\ \angle 30°$

$\mathbf{E}_2 = 20\text{ V}\ \angle 0°$

7. a.



$\mathbf{Z}_1 = 12\ \Omega + j12\ \Omega = 16.971\ \Omega \angle 45°$
$\mathbf{Z}_2 = 3\ \Omega \angle 0°$
$\mathbf{Z}_3 = -j1\ \Omega$
$\mathbf{E}_1 = 20\ \text{V} \angle 50°$
$\mathbf{E}_2 = 60\ \text{V} \angle 70°$
$\mathbf{E}_3 = 40\ \text{V} \angle 0°$

$$\mathbf{I}_1[\mathbf{Z}_1 + \mathbf{Z}_2] - \mathbf{Z}_2\mathbf{I}_2 = \mathbf{E}_1 - \mathbf{E}_2$$
$$\mathbf{I}_2[\mathbf{Z}_2 + \mathbf{Z}_3] - \mathbf{Z}_2\mathbf{I}_1 = \mathbf{E}_2 - \mathbf{E}_3$$

$$(\mathbf{Z}_1 + \mathbf{Z}_2)\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 = \mathbf{E}_1 - \mathbf{E}_2$$
$$-\mathbf{Z}_2\mathbf{I}_1 + (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_2 = \mathbf{E}_2 - \mathbf{E}_3$$

Using determinants:

$$\mathbf{I}_{R_1} = \mathbf{I}_1 = \frac{(\mathbf{E}_1 - \mathbf{E}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) + \mathbf{Z}_2(\mathbf{E}_2 - \mathbf{E}_3)}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3} = \mathbf{2.55\ A \angle 132.72°}$$

8. Clockwise mesh currents:

$$\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2\mathbf{Z}_2 = 0$$
$$-\mathbf{I}_2\mathbf{Z}_2 + \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{I}_2\mathbf{Z}_4 + \mathbf{I}_3\mathbf{Z}_4 = 0$$
$$-\mathbf{I}_3\mathbf{Z}_4 + \mathbf{I}_2\mathbf{Z}_4 - \mathbf{I}_3\mathbf{Z}_5 - \mathbf{E}_2 = 0$$

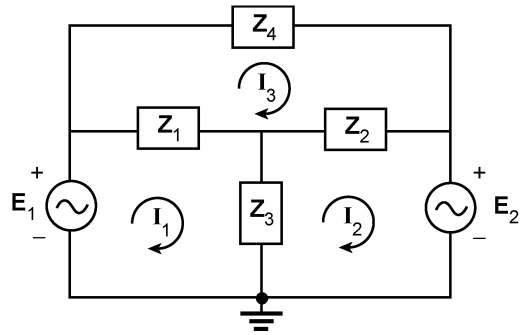
$$[\mathbf{Z}_1 + \mathbf{Z}_2]\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 + 0 = \mathbf{E}_1$$
$$-\mathbf{Z}_2\mathbf{I}_1 + [\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4]\mathbf{I}_2 - \mathbf{Z}_4\mathbf{I}_3 = 0$$
$$0 - \mathbf{Z}_4\mathbf{I}_2 + [\mathbf{Z}_4 + \mathbf{Z}_5]\mathbf{I}_3 = -\mathbf{E}_2$$

$$X_{L_1} = \omega L_1 = (2\pi)(2\ \text{kHz})(110\ \mu\text{H}) = 1.38\ \Omega$$
$$X_{L_2} = \omega L_2 = (2\pi)(2\ \text{kHz})(220\ \mu\text{H}) = 2.76\ \Omega$$
$$X_{C_1} = \frac{1}{\omega C_1} = \frac{1}{2\pi(2\ \text{kHz})(39\mu\text{F})} = 3.62\ \Omega$$
$$X_{C_2} = \frac{1}{\omega C_2} = \frac{1}{2\pi(2\ \text{kHz})(39\mu\text{F})} = 2.04\ \Omega$$

$\mathbf{Z}_1 = 4\ \Omega + j1.38\ \Omega$
$\mathbf{Z}_2 = -j3.62\ \Omega$
$\mathbf{Z}_3 = j2.76\ \Omega$
$\mathbf{Z}_4 = -j2.04\ \Omega$
$\mathbf{Z}_5 = 8\ \Omega \angle 0°$
$\mathbf{E}_1 = 6\ \text{V} \angle 0°$
$\mathbf{E}_2 = 120\ \text{V} \angle 120°$

$$\mathbf{I}_{R_1} = \mathbf{I}_3 = \frac{[\mathbf{Z}_2\mathbf{Z}_4]\mathbf{E}_1 + \left[\mathbf{Z}_2^2 - [\mathbf{Z}_1 + \mathbf{Z}_2][\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4]\right]\mathbf{E}_2}{[\mathbf{Z}_1 + \mathbf{Z}_2][\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4][\mathbf{Z}_4 + \mathbf{Z}_5] - [\mathbf{Z}_1 + \mathbf{Z}_2]\mathbf{Z}_4^2 - [\mathbf{Z}_4 + \mathbf{Z}_5]\mathbf{Z}_2^2}$$
$$= \frac{1000\ \text{V} \angle -64.5°}{124.4\ \Omega \angle -68.34°} = \mathbf{8.04\ A \angle 3.84°}$$

9.



$\mathbf{Z}_1 = 15\ \Omega\ \angle 0°$, $\mathbf{Z}_2 = 15\ \Omega\ \angle 0°$
$\mathbf{Z}_3 = -j10\ \Omega = 10\ \Omega\ \angle -90°$
$\mathbf{Z}_4 = 3\ \Omega + j4\ \Omega = 5\ \Omega\ \angle 53.13°$
$\mathbf{E}_1 = 220\ \text{V}\ \angle 0°$
$\mathbf{E}_2 = 100\ \text{V}\ \angle 90°$

$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_3) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{I}_3\mathbf{Z}_1 = \mathbf{E}_1$
$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_3 - \mathbf{I}_3\mathbf{Z}_2 = -\mathbf{E}_2$
$\mathbf{I}_3(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_4) - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2 = 0$

$$\begin{aligned}
\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_3) &- \mathbf{I}_2\mathbf{Z}_3 &- \mathbf{I}_3\mathbf{Z}_1 &= \mathbf{E}_1\\
-\mathbf{I}_1\mathbf{Z}_3 &+ \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) &- \mathbf{I}_3\mathbf{Z}_2 &= -\mathbf{E}_2\\
-\mathbf{I}_1\mathbf{Z}_1 &- \mathbf{I}_2\mathbf{Z}_2 &+ \mathbf{I}_3(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_4) &= 0
\end{aligned}$$

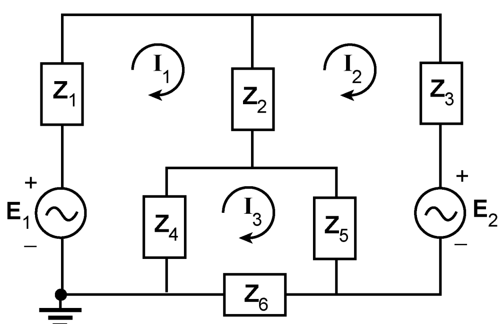
Applying determinants:

$$\mathbf{I}_3 = \frac{-(\mathbf{Z}_1+\mathbf{Z}_3)(\mathbf{Z}_2)\mathbf{E}_2 - \mathbf{Z}_1\mathbf{Z}_3\mathbf{E}_2 + \mathbf{E}_1[\mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1(\mathbf{Z}_2+\mathbf{Z}_3)]}{(\mathbf{Z}_1+\mathbf{Z}_3)[(\mathbf{Z}_2+\mathbf{Z}_3)(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_4) - \mathbf{Z}_2^2] + \mathbf{Z}_3[\mathbf{Z}_3(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_4) - \mathbf{Z}_1\mathbf{Z}_2] - \mathbf{Z}_1[-\mathbf{Z}_2\mathbf{Z}_3 - \mathbf{Z}_1(\mathbf{Z}_2+\mathbf{Z}_3)]}$$

$= \mathbf{48.33\ A\ \angle{-77.57°}}$

or $\mathbf{I}_3 = \dfrac{\mathbf{E}_1 - \mathbf{E}_2}{\mathbf{Z}_4}$ if one carefully examines the network!

10.



$\mathbf{Z}_1 = 5\ \Omega\ \angle 0°$, $\mathbf{Z}_2 = 5\ \Omega\ \angle 90°$
$\mathbf{Z}_3 = 4\ \Omega\ \angle 0°$, $\mathbf{Z}_4 = 6\ \Omega\ \angle -90°$
$\mathbf{Z}_5 = 4\ \Omega\ \angle 0°$, $\mathbf{Z}_6 = 6\ \Omega + j8\ \Omega$
$\mathbf{E}_1 = 20\ \text{V}\ \angle 0°$, $\mathbf{E}_2 = 40\ \text{V}\ \angle 60°$

$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_4 = \mathbf{E}_1$
$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_5) - \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_5 = -\mathbf{E}_2$
$\mathbf{I}_3(\mathbf{Z}_4 + \mathbf{Z}_5 + \mathbf{Z}_6) - \mathbf{I}_1\mathbf{Z}_4 - \mathbf{I}_2\mathbf{Z}_5 = 0$

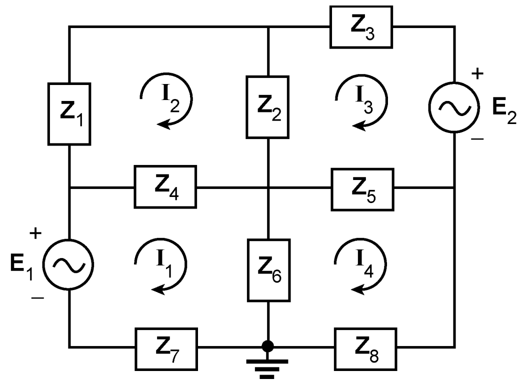
$$\begin{aligned}
(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_4)\,\mathbf{I}_1 &- \mathbf{Z}_2\mathbf{I}_2 &- \mathbf{Z}_4\mathbf{I}_3 &= \mathbf{E}_1\\
-\mathbf{Z}_2\mathbf{I}_1 &+ (\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_5)\mathbf{I}_2 &- \mathbf{Z}_5\mathbf{I}_3 &= -\mathbf{E}_2\\
-\mathbf{Z}_4\mathbf{I}_1 &- \mathbf{Z}_5\mathbf{I}_2 &+ (\mathbf{Z}_4 + \mathbf{Z}_5 + \mathbf{Z}_6)\mathbf{I}_3 &= 0
\end{aligned}$$

Using $\mathbf{Z}' = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_4$, $\mathbf{Z}'' = \mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_5$, $\mathbf{Z}''' = \mathbf{Z}_4 + \mathbf{Z}_5 + \mathbf{Z}_6$ and determinants:

$$\mathbf{I}_{R_1} = \mathbf{I}_1 = \frac{\mathbf{E}_1(\mathbf{Z}''\mathbf{Z}''' - \mathbf{Z}_5^2) - \mathbf{E}_2(\mathbf{Z}_2\mathbf{Z}''' + \mathbf{Z}_4\mathbf{Z}_5)}{\mathbf{Z}'(\mathbf{Z}''\mathbf{Z}''' - \mathbf{Z}_5^2) - \mathbf{Z}_2(\mathbf{Z}_2\mathbf{Z}''' + \mathbf{Z}_4\mathbf{Z}_5) - \mathbf{Z}_4(\mathbf{Z}_2\mathbf{Z}_5 + \mathbf{Z}_4\mathbf{Z}'')}$$

$= \mathbf{3.04\ A\ \angle 169.12°}$

11.



$\mathbf{Z}_1 = 10\ \Omega + j20\ \Omega$ $\quad\mathbf{Z}_2 = -j20\ \Omega$
$\mathbf{Z}_3 = 80\ \Omega \angle 0°$ $\quad\mathbf{Z}_4 = 6\ \Omega \angle 0°$
$\mathbf{Z}_5 = 15\ \Omega \angle 90°$ $\quad\mathbf{Z}_6 = 10\ \Omega \angle 0°$
$\mathbf{Z}_7 = 5\ \Omega \angle 0°$ $\quad\mathbf{Z}_8 = 5\ \Omega - j20\ \Omega$
$\mathbf{E}_1 = 25\ \text{V} \angle 0°$ $\quad\mathbf{E}_2 = 75\ \text{V} \angle 20°$

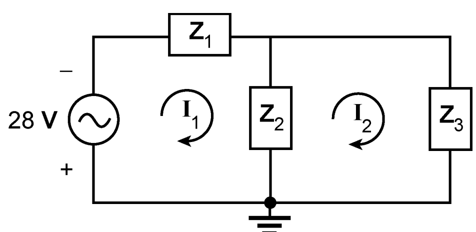
$\mathbf{I}_1(\mathbf{Z}_4 + \mathbf{Z}_6 + \mathbf{Z}_7) - \mathbf{I}_2\mathbf{Z}_4 - \mathbf{I}_4\mathbf{Z}_6 = \mathbf{E}_1$
$\mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_4) - \mathbf{I}_1\mathbf{Z}_4 - \mathbf{I}_3\mathbf{Z}_2 = 0$
$\mathbf{I}_3(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_5) - \mathbf{I}_2\mathbf{Z}_2 - \mathbf{I}_4\mathbf{Z}_5 = -\mathbf{E}_2$
$\mathbf{I}_4(\mathbf{Z}_5 + \mathbf{Z}_6 + \mathbf{Z}_8) - \mathbf{I}_1\mathbf{Z}_6 - \mathbf{I}_3\mathbf{Z}_5 = 0$

$(\mathbf{Z}_4 + \mathbf{Z}_6 + \mathbf{Z}_7)\,\mathbf{I}_1 \quad - \mathbf{Z}_4\mathbf{I}_2 \quad + 0 \quad - \mathbf{Z}_6\mathbf{I}_4 = \mathbf{E}_1$
$-\mathbf{Z}_4\mathbf{I}_1 + (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_4)\mathbf{I}_2 \quad - \mathbf{Z}_2\mathbf{I}_3 \quad + 0 = 0$
$0 \quad - \mathbf{Z}_2\ \mathbf{I}_2 + (\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_5)\mathbf{I}_3 \quad - \mathbf{Z}_5\mathbf{I}_4 = -\mathbf{E}_2$
$-\mathbf{Z}_6\mathbf{I}_1 \quad + 0 \quad - \mathbf{Z}_5\mathbf{I}_3 + (\mathbf{Z}_5 + \mathbf{Z}_6 + \mathbf{Z}_7)\mathbf{I}_4 = 0$

Applying determinants:

$\mathbf{I}_{R_1} = \mathbf{I}_{80\Omega} = \mathbf{0.68\ A \angle{-162.9°}}$

12.



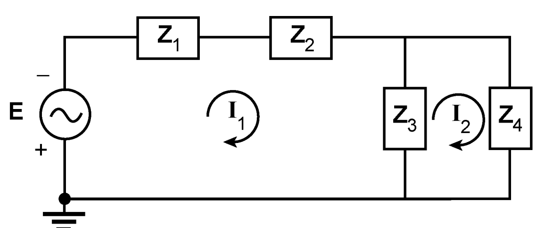
$\mathbf{Z}_1 = 5\ \text{k}\Omega \angle 0°$
$\mathbf{Z}_2 = 10\ \text{k}\Omega \angle 0°$
$\mathbf{Z}_3 = 1\ \text{k}\Omega + j4\ \text{k}\Omega = 4.123\ \text{k}\Omega \angle 75.96°$

$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{Z}_2\mathbf{I}_2 = -28\ \mathbf{V}$
$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{Z}_2\mathbf{I}_1 = 0$

$(\mathbf{Z}_1 + \mathbf{Z}_2)\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 = -28\ \mathbf{V}$
$-\mathbf{Z}_2\mathbf{I}_1 + (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_2 = 0$

$$\mathbf{I}_L = \mathbf{I}_2 = \frac{-\mathbf{Z}_2\,28\ \mathbf{V}}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3} = \mathbf{-3.17 \times 10^{-3}\ V \angle 137.29°}$$

13.



Source Conversion:

$\mathbf{E} = (I \angle \theta)(R_p \angle 0°)$
$= (50\ \mathbf{I})(40\ \text{k}\Omega \angle 0°)$
$= 2 \times 10^6\ \mathbf{I} \angle 0°$
$\mathbf{Z}_1 = R_s = R_p = 40\ \text{k}\Omega \angle 0°$
$\mathbf{Z}_2 = -j0.2\ \text{k}\Omega$
$\mathbf{Z}_3 = 8\ \text{k}\Omega \angle 0°$
$\mathbf{Z}_4 = 4\ \text{k}\Omega \angle 90°$

$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{Z}_3\mathbf{I}_2 = -\mathbf{E}$
$\mathbf{I}_2(\mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{Z}_3\mathbf{I}_1 = 0$

$(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_1 - \mathbf{Z}_3\mathbf{I}_2 = -\mathbf{E}$
$-\mathbf{Z}_3\mathbf{I}_1 + (\mathbf{Z}_3 + \mathbf{Z}_4)\mathbf{I}_2 = 0$

$$\mathbf{I}_L = \mathbf{I}_2 = \frac{-\mathbf{Z}_3\mathbf{E}}{(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3)(\mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{Z}_3^2} = \mathbf{42.91\ I\ \angle 149.31^\circ}$$

14. $6\mathbf{V}_x - \mathbf{I}_1\ 1\text{ k}\Omega - 10\text{ V}\angle 0^\circ = 0$
$10\text{ V}\angle 0^\circ - \mathbf{I}_2\ 4\text{ k}\Omega - \mathbf{I}_2\ 2\text{ k}\Omega = 0$

$\mathbf{V}_x = \mathbf{I}_2\ 2\text{ k}\Omega$

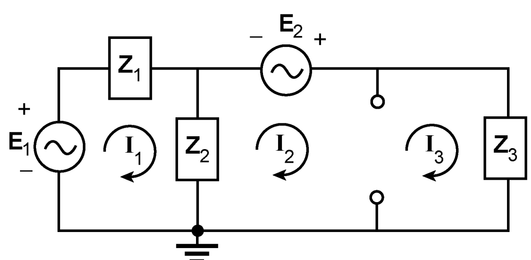
$-\mathbf{I}_1\ 1\text{ k}\Omega + \mathbf{I}_2\ 12\text{ k}\Omega = 10\text{ V}\angle 0^\circ$
$-\mathbf{I}_2\ 6\text{ k}\Omega = -10\text{ V}\angle 0^\circ$

$$\mathbf{I}_2 = \mathbf{I}_{2\text{k}\Omega} = \frac{10\text{ V}\angle 0^\circ}{6\text{ k}\Omega} = \mathbf{1.67\ mA\ \angle 0^\circ} = \mathbf{I}_{2\text{k}\Omega}$$

$-\mathbf{I}_1\ 1\text{ k}\Omega + (1.667\text{ mA}\angle 0^\circ)(12\text{ k}\Omega) = 10\text{ V}\angle 0^\circ$
$-\mathbf{I}_1\ 1\text{ k}\Omega + 20\text{ V}\angle 0^\circ = 10\text{ V}\angle 0^\circ$
$-\mathbf{I}_1\ 1\text{ k}\Omega = -10\text{ V}\angle 0^\circ$

$$\mathbf{I}_1 = \mathbf{I}_{1\text{k}\Omega} = \frac{10\text{ V}\angle 0^\circ}{1\text{ k}\Omega} = \mathbf{10\ mA\ \angle 0^\circ}$$

15.



$\mathbf{E}_1 = 5\text{ V}\angle 0^\circ$
$\mathbf{E}_2 = 20\text{ V}\angle 0^\circ$
$\mathbf{Z}_1 = 2.2\text{ k}\Omega\angle 0^\circ$
$\mathbf{Z}_2 = 5\text{ k}\Omega\angle 90^\circ$
$\mathbf{Z}_3 = 10\text{ k}\Omega\angle 0^\circ$
$\mathbf{I} = 4\text{ mA}\angle 0^\circ$

$\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{Z}_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$
$-\mathbf{Z}_2(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{E}_2 - \mathbf{I}_3\mathbf{Z}_3 = 0$

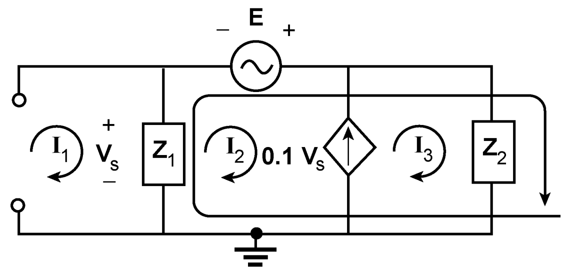
$\mathbf{I}_3 - \mathbf{I}_2 = \mathbf{I}$

Substituting, we obtain:

$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1$
$\mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) = \mathbf{I}\mathbf{Z}_3 - \mathbf{E}_2$

Determinants:

$\mathbf{I}_1 = 1.39\text{ mA}\angle -126.48^\circ$, $\mathbf{I}_2 = 1.341\text{ mA}\angle -10.56^\circ$, $\mathbf{I}_3 = 2.693\text{ mA}\angle -174.8^\circ$
$\mathbf{I}_{10\text{k}\Omega} = \mathbf{I}_3 = \mathbf{2.69\ mA\ \angle -174.8^\circ}$

16. 

$\mathbf{Z}_1 = 1\ \text{k}\Omega\ \angle 0°$
$\mathbf{Z}_2 = 4\ \text{k}\Omega + j6\ \text{k}\Omega$
$\mathbf{E} = 10\ \text{V}\ \angle 0°$

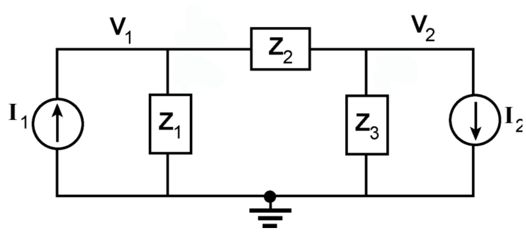
$-\mathbf{Z}_1(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{E} - \mathbf{I}_3\mathbf{Z}_3 = 0$
$\mathbf{I}_1 = 6\ \text{mA}\ \angle 0°,\ 0.1\ \mathbf{V}_s = \mathbf{I}_3 - \mathbf{I}_2,\ \mathbf{V}_s = (\mathbf{I}_1 - \mathbf{I}_2)\mathbf{Z}_1$

Substituting:

$(1\ \text{k}\Omega)\mathbf{I}_2 + (4\ \text{k}\Omega + j6\ \text{k}\Omega)\mathbf{I}_3 = 16\ \text{V}\ \angle 0°$
$(99\ \Omega)\mathbf{I}_2 + \qquad \mathbf{I}_3 = 0.6\ \text{V}\ \angle 0°$

Determinants:

$\mathbf{I}_3 = \mathbf{I}_{6\ \text{k}\Omega} = \mathbf{1.38\ mA\ \angle -56.31°}$

17. 

$\mathbf{Z}_1 = 4\ \text{k}\Omega\ \angle 0°$
$\mathbf{Z}_2 = 1\ \text{k}\Omega\ \angle 90°$
$\mathbf{Z}_3 = 8\ \text{k}\Omega\ \angle -90°$
$\mathbf{I}_1 = 3\ \text{mA}\ \angle 0°$
$\mathbf{I}_2 = 5\ \text{mA}\ \angle 30°$

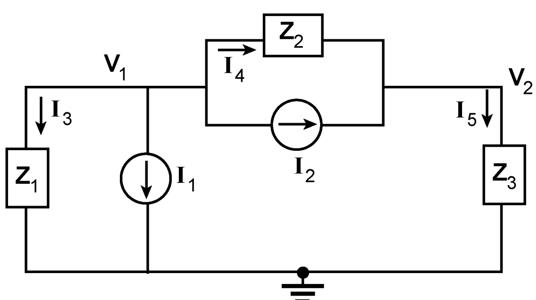
$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right] - \frac{1}{\mathbf{Z}_2}\mathbf{V}_2 = \mathbf{I}_1$$
$$\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] - \frac{1}{\mathbf{Z}_2}\mathbf{V}_1 = -\mathbf{I}_2$$

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_2}\right] = \mathbf{I}_1$$
$$-\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_2}\right] + \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] = -\mathbf{I}_2$$

$$\mathbf{V}_1 = \frac{\begin{vmatrix} \mathbf{I}_1 & -\dfrac{1}{\mathbf{Z}_2} \\ -\mathbf{I}_2 & \left(\dfrac{1}{\mathbf{Z}_2}+\dfrac{1}{\mathbf{Z}_3}\right) \end{vmatrix}}{\begin{vmatrix} \left(\dfrac{1}{\mathbf{Z}_1}+\dfrac{1}{\mathbf{Z}_2}\right) & -\dfrac{1}{\mathbf{Z}_2} \\ -\dfrac{1}{\mathbf{Z}_2} & \left(\dfrac{1}{\mathbf{Z}_2}+\dfrac{1}{\mathbf{Z}_3}\right) \end{vmatrix}} = \mathbf{11.99\ V \angle -154.53^\circ}$$

$$\mathbf{V}_2 = \frac{\begin{vmatrix} \left(\dfrac{1}{\mathbf{Z}_1}+\dfrac{1}{\mathbf{Z}_2}\right) & \mathbf{I}_1 \\ -\dfrac{1}{\mathbf{Z}_2} & -\mathbf{I}_2 \end{vmatrix}}{\begin{vmatrix} \quad '' \quad \end{vmatrix}} = \mathbf{14.46\ V \angle -131.28^\circ}$$

18.



$\mathbf{Z}_1 = 3\ \Omega + j4\ \Omega = 5\ \Omega \angle 53.13^\circ$
$\mathbf{Z}_2 = 2\ \Omega \angle 0^\circ$
$\mathbf{Z}_3 = 6\ \Omega \angle 0^\circ \parallel 8\ \Omega \angle -90^\circ$
$\quad = 4.8\ \Omega \angle -36.87^\circ$
$\mathbf{I}_1 = 0.6\ \text{A} \angle 20^\circ$
$\mathbf{I}_2 = 4\ \text{A} \angle 80^\circ$

$0 = \mathbf{I}_1 + \mathbf{I}_3 + \mathbf{I}_4 + \mathbf{I}_2$

$$0 = \mathbf{I}_1 + \frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2} + \mathbf{I}_2$$

$$\mathbf{V}_1 \left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right] - \mathbf{V}_2 \left[\frac{1}{\mathbf{Z}_2}\right] = -\mathbf{I}_1 - \mathbf{I}_2$$

or $\quad \underline{\mathbf{V}_1[\mathbf{Y}_1 + \mathbf{Y}_2] - \mathbf{V}_2[\mathbf{Y}_2] = -\mathbf{I}_1 - \mathbf{I}_2}$

$\mathbf{I}_2 + \mathbf{I}_4 = \mathbf{I}_5$

$$\mathbf{I}_2 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2} = \frac{\mathbf{V}_2}{\mathbf{Z}_3}$$

$$\mathbf{V}_2 \left[\frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] - \mathbf{V}_1 \left[\frac{1}{\mathbf{Z}_2}\right] = +\mathbf{I}_2$$

or $\quad \underline{\mathbf{V}_2[\mathbf{Y}_2 + \mathbf{Y}_3] - \mathbf{V}_1[\mathbf{Y}_2] = \mathbf{I}_2}$

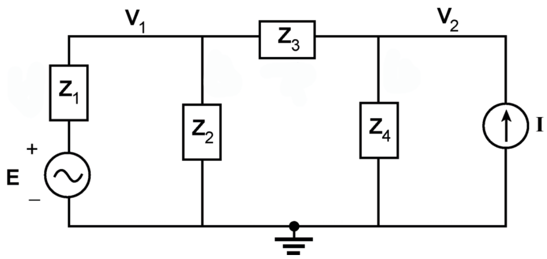
and $\quad [\mathbf{Y}_1 + \mathbf{Y}_2]\mathbf{V}_1 - \mathbf{Y}_2\mathbf{V}_2 = -\mathbf{I}_1 - \mathbf{I}_2$
$\quad -\mathbf{Y}_2\mathbf{V}_1 + [\mathbf{Y}_2 + \mathbf{Y}_3]\mathbf{V}_2 = \mathbf{I}_2$

Applying determinants:

$$\mathbf{V}_1 = \frac{-[\mathbf{Y}_2+\mathbf{Y}_3][\mathbf{I}_1+\mathbf{I}_2]+\mathbf{Y}_2\mathbf{I}_2}{\mathbf{Y}_1\mathbf{Y}_2+\mathbf{Y}_1\mathbf{Y}_3+\mathbf{Y}_2\mathbf{Y}_3} = \mathbf{5.12\ V\ \angle{-79.36^\circ}}$$

$$\mathbf{V}_2 = \frac{\mathbf{Y}_1\mathbf{I}_2-\mathbf{I}_1\mathbf{Y}_2}{\mathbf{Y}_1\mathbf{Y}_2+\mathbf{Y}_1\mathbf{Y}_3+\mathbf{Y}_2\mathbf{Y}_3} = \mathbf{2.71\ V\ \angle 39.96^\circ}$$

19.



$\mathbf{Z}_1 = 5\ \Omega\ \angle 0^\circ$
$\mathbf{Z}_4 = 2\ \Omega\ \angle 0^\circ$
$\mathbf{E} = 30\ \text{V}\ \angle 50^\circ$
$\mathbf{I} = 4\ \text{A}\ \angle 90^\circ$
$X_L = 2\pi f L = 2\pi(10\text{ kHz})(0.1\text{ mH}) = 6.28\Omega$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(10\text{ kHz})(4.7\ \mu\text{F})} = 3.39\ \Omega$$

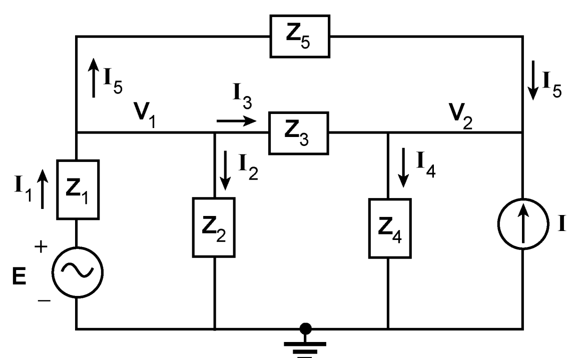
$\mathbf{Z}_2 = 6.28\ \Omega\ \angle 90^\circ$
$\mathbf{Z}_3 = 3.39\ \Omega\ \angle{-90^\circ}$

$$\left.\begin{aligned}\mathbf{V}_1[\mathbf{Y}_1+\mathbf{Y}_2+\mathbf{Y}_3]-\mathbf{V}_2\mathbf{Y}_3 &= \mathbf{E}_1\mathbf{Y}_1\\ -\mathbf{V}_1[\mathbf{Y}_3]+\mathbf{V}_2[\mathbf{Y}_3+\mathbf{Y}_4] &= +\mathbf{I}\end{aligned}\right\}\text{ after source conversion.}$$

Using determinants:

$\mathbf{V}_1 = \mathbf{17.92\ V\ \angle 59.25^\circ}$ and $\mathbf{V}_2 = \mathbf{13.95\ V\ \angle 93.64^\circ}$

20.



$\mathbf{Z}_1 = 10\ \Omega\ \angle 0^\circ$
$\mathbf{Z}_2 = 10\ \Omega\ \angle 0^\circ$
$\mathbf{Z}_3 = 4\ \Omega\ \angle 90^\circ$
$\mathbf{Z}_4 = 2\ \Omega\ \angle 0^\circ$
$\mathbf{Z}_5 = 8\ \Omega\ \angle{-90^\circ}$
$\mathbf{E} = 50\ \text{V}\ \angle 120^\circ$
$\mathbf{I} = 0.8\ \text{A}\ \angle 70^\circ$

$\mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_5$

$$\frac{\mathbf{E}-\mathbf{V}_1}{\mathbf{Z}_1} = \frac{\mathbf{V}_1}{\mathbf{Z}_2} + \frac{(\mathbf{V}_1-\mathbf{V}_2)}{\mathbf{Z}_5} + \frac{\mathbf{V}_1-\mathbf{V}_2}{\mathbf{Z}_3} \Rightarrow \mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_5}\right]-\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_5}\right]=\frac{\mathbf{E}}{\mathbf{Z}_1}$$

or $\mathbf{V}_1[\mathbf{Y}_1+\mathbf{Y}_2+\mathbf{Y}_3+\mathbf{Y}_5]-\mathbf{V}_2[\mathbf{Y}_3+\mathbf{Y}_5] = \mathbf{E}_1\mathbf{Y}_1$

$\mathbf{I}_3 + \mathbf{I}_5 = \mathbf{I}_4 + \mathbf{I}$

$$\frac{\mathbf{V}_1-\mathbf{V}_2}{\mathbf{Z}_3}+\frac{\mathbf{V}_1-\mathbf{V}_2}{\mathbf{Z}_5}=\frac{\mathbf{V}_2}{\mathbf{Z}_4}+\mathbf{I} \Rightarrow \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4}+\frac{1}{\mathbf{Z}_5}\right]-\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_5}\right]=-\mathbf{I}$$

or $\mathbf{V}_2[\mathbf{Y}_3+\mathbf{Y}_4+\mathbf{Y}_5]-\mathbf{V}_1[\mathbf{Y}_3+\mathbf{Y}_5] = -\mathbf{I}$

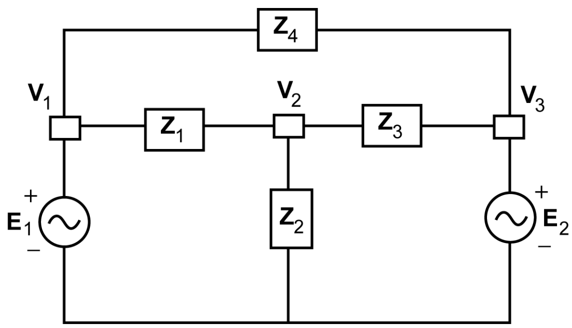
resulting in

$$\mathbf{V}_1[\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_5] - \mathbf{V}_2[\mathbf{Y}_3 + \mathbf{Y}_5] = \mathbf{E}_1\mathbf{Y}_1$$
$$-\mathbf{V}_1[\mathbf{Y}_3 + \mathbf{Y}_5] + \mathbf{V}_2[\mathbf{Y}_3 + \mathbf{Y}_4 + \mathbf{Y}_5] = -\mathbf{I}$$

Applying determinants:

$\mathbf{V}_1$ = **19.78 V ∠132.48°** and $\mathbf{V}_2$ = **13.37 V ∠98.78°**

21.



$\mathbf{Z}_1 = 15\ \Omega\ \angle 0°$
$\mathbf{Z}_2 = 10\ \Omega\ \angle -90°$
$\mathbf{Z}_3 = 15\ \Omega\ \angle 0°$
$\mathbf{Z}_4 = 3\ \Omega + j4\ \Omega$

$$\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] - \frac{1}{\mathbf{Z}_1}\mathbf{V}_1 - \frac{1}{\mathbf{Z}_3}\mathbf{V}_3 = 0$$

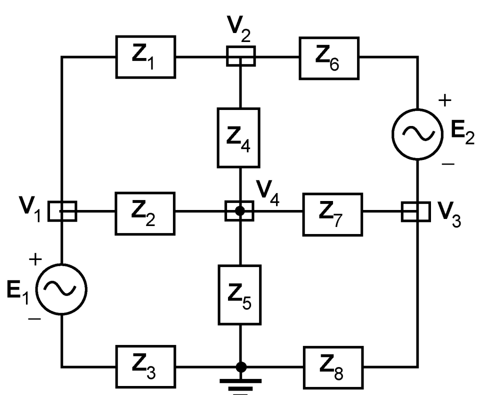
$$\mathbf{V}_2\left[\frac{1}{15\ \Omega} + \frac{1}{-j10\ \Omega} + \frac{1}{15\ \Omega}\right] - \frac{1}{15\ \Omega}\left[220\ \text{V} \angle 0°\right] - \frac{1}{15\ \Omega}\left[100\ \text{V} \angle 90°\right] = 0$$

$$\mathbf{V}_2\ [133.34 \times 10^{-3} + j100 \times 10^{-3}] = 14.67 + j6.67$$

$$\mathbf{V}_2 = \frac{16.05\ \text{V} \angle 24.55°}{166.67 \times 10^{-3} \angle 36.37°} = \mathbf{96.30\ V \angle{-12.32°}}$$

$\mathbf{V}_1$ = $\mathbf{E}_1$ = **220 V ∠0°**, $\mathbf{V}_3$ = $\mathbf{E}_2$ = **100 V ∠90°**

22.



$\mathbf{E}_1 = 25\ \text{V}\ \angle 0°$
$\mathbf{E}_2 = 75\ \text{V}\ \angle 20°$

$\mathbf{Z}_1 = 10\ \Omega + j20\ \Omega$
$\mathbf{Z}_2 = 6\ \Omega\ \angle 0°$
$\mathbf{Z}_3 = 5\ \Omega\ \angle 0°$
$\mathbf{Z}_4 = 20\ \Omega\ \angle -90°$
$\mathbf{Z}_5 = 10\ \Omega\ \angle 0°$
$\mathbf{Z}_6 = 80\ \Omega\ \angle 0°$
$\mathbf{Z}_7 = 15\ \Omega\ \angle 90°$
$\mathbf{Z}_8 = 5\ \Omega - j20\ \Omega$

$$\mathbf{V}_1:\ \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_1} + \frac{\mathbf{V}_1 - \mathbf{V}_4}{\mathbf{Z}_2} + \frac{\mathbf{V}_1 - \mathbf{E}_1}{\mathbf{Z}_3} = 0$$

$$\mathbf{V}_2:\ \frac{\mathbf{V}_2 - \mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_2 - \mathbf{V}_4}{\mathbf{Z}_4} + \frac{\mathbf{V}_2 - \mathbf{E}_2 - \mathbf{V}_3}{\mathbf{Z}_6} = 0$$

$$\mathbf{V}_3:\ \frac{\mathbf{V}_3 + \mathbf{E}_2 - \mathbf{V}_2}{\mathbf{Z}_6} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{\mathbf{Z}_7} + \frac{\mathbf{V}_3}{\mathbf{Z}_8} = 0$$

$$\mathbf{V}_4:\ \frac{\mathbf{V}_4 - \mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_4 - \mathbf{V}_2}{\mathbf{Z}_4} + \frac{\mathbf{V}_4 - \mathbf{V}_3}{\mathbf{Z}_7} + \frac{\mathbf{V}_4}{\mathbf{Z}_5} = 0$$

Rearranging:

$$\mathbf{V}_1\left(\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}\right)-\frac{\mathbf{V}_2}{\mathbf{Z}_1}-\frac{\mathbf{V}_4}{\mathbf{Z}_2}=\frac{\mathbf{E}_1}{\mathbf{Z}_3}$$

$$\mathbf{V}_2\left(\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_4}+\frac{1}{\mathbf{Z}_6}\right)-\frac{\mathbf{V}_1}{\mathbf{Z}_1}-\frac{\mathbf{V}_4}{\mathbf{Z}_4}-\frac{\mathbf{V}_3}{\mathbf{Z}_6}=\frac{\mathbf{E}_2}{\mathbf{Z}_6}$$

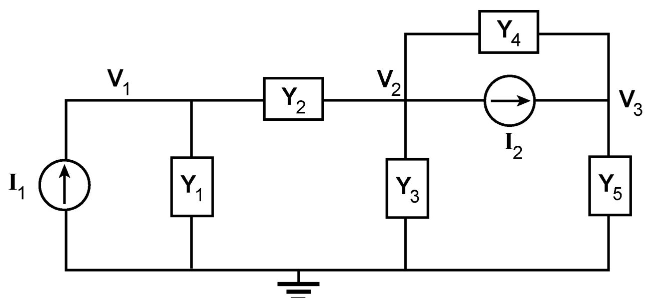
$$\mathbf{V}_3\left(\frac{1}{\mathbf{Z}_6}+\frac{1}{\mathbf{Z}_7}+\frac{1}{\mathbf{Z}_8}\right)-\frac{\mathbf{V}_2}{\mathbf{Z}_6}-\frac{\mathbf{V}_4}{\mathbf{Z}_7}=-\frac{\mathbf{E}_2}{\mathbf{Z}_6}$$

$$\mathbf{V}_4\left(\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_4}+\frac{1}{\mathbf{Z}_7}+\frac{1}{\mathbf{Z}_5}\right)-\frac{\mathbf{V}_1}{\mathbf{Z}_2}-\frac{\mathbf{V}_2}{\mathbf{Z}_4}-\frac{\mathbf{V}_3}{\mathbf{Z}_7}=0$$

Setting up and then using determinants:

$$\mathbf{V}_1 = \mathbf{14.62\ V\ \angle{-5.86^\circ}},\ \mathbf{V}_2 = \mathbf{35.03\ V\ \angle{-37.69^\circ}}$$
$$\mathbf{V}_3 = \mathbf{32.4\ V\ \angle{-73.34^\circ}},\ \mathbf{V}_4 = \mathbf{5.67\ V\ \angle 23.53^\circ}$$

23.



$$\mathbf{Y}_1 = \frac{1}{4\ \Omega\angle 0^\circ} = 0.25\ \text{S}\ \angle 0^\circ$$

$$\mathbf{Y}_2 = \frac{1}{1\ \Omega\angle 90^\circ} = 1\ \text{S}\ \angle{-90^\circ}$$

$$\mathbf{Y}_3 = \frac{1}{5\ \Omega\angle 0^\circ} = 0.2\ \text{S}\ \angle 0^\circ$$

$$\mathbf{Y}_4 = \frac{1}{4\ \Omega\angle{-90^\circ}} = 0.25\ \text{S}\ \angle 90^\circ$$

$$\mathbf{Y}_5 = \frac{1}{8\ \Omega\angle 90^\circ} = 0.125\ \text{S}\ \angle{-90^\circ}$$

$\mathbf{I}_1 = 2\ \text{A}\ \angle 30^\circ$

$\mathbf{I}_2 = 3\ \text{A}\ \angle 150^\circ$

$$\mathbf{V}_1[\mathbf{Y}_1+\mathbf{Y}_2]-\mathbf{Y}_2\mathbf{V}_2=\mathbf{I}_1$$
$$\mathbf{V}_2[\mathbf{Y}_2+\mathbf{Y}_3+\mathbf{Y}_4]-\mathbf{Y}_2\mathbf{V}_1-\mathbf{Y}_4\mathbf{V}_3=-\mathbf{I}_2$$
$$\mathbf{V}_3[\mathbf{Y}_4+\mathbf{Y}_5]-\mathbf{Y}_4\mathbf{V}_2=\mathbf{I}_2$$

$$\begin{aligned}
&[\mathbf{Y}_1+\mathbf{Y}_2]\mathbf{V}_1 && -\mathbf{Y}_2\mathbf{V}_2 && +0 &&= \mathbf{I}_1\\
&-\mathbf{Y}_2\mathbf{V}_1 && +[\mathbf{Y}_2+\mathbf{Y}_3+\mathbf{Y}_4]\mathbf{V}_2 && -\mathbf{Y}_4\mathbf{V}_3 &&= -\mathbf{I}_2\\
&0 && -\mathbf{Y}_4\mathbf{V}_2 && +[\mathbf{Y}_4+\mathbf{Y}_5]\mathbf{V}_3 &&= \mathbf{I}_2
\end{aligned}$$

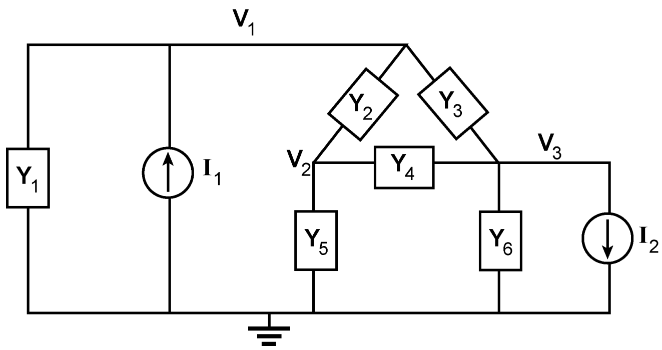
$$\mathbf{V}_1 = \frac{\mathbf{I}_1\left[(\mathbf{Y}_2+\mathbf{Y}_3+\mathbf{Y}_4)(\mathbf{Y}_4+\mathbf{Y}_5)-\mathbf{Y}_4^2\right]-\mathbf{I}_2[\mathbf{Y}_2\mathbf{Y}_5]}{[\mathbf{Y}_1+\mathbf{Y}_2]\left[(\mathbf{Y}_2+\mathbf{Y}_3+\mathbf{Y}_4)(\mathbf{Y}_4+\mathbf{Y}_5)-\mathbf{Y}_4^2\right]-\mathbf{Y}_2^2(\mathbf{Y}_4+\mathbf{Y}_5)=\mathbf{Y}_\Delta}$$

$$= \mathbf{5.74\ V\ \angle 122.76^\circ}$$

$$\mathbf{V}_2 = \frac{\mathbf{I}_1\mathbf{Y}_2(\mathbf{Y}_4+\mathbf{Y}_5) - \mathbf{I}_2\mathbf{Y}_5(\mathbf{Y}_1+\mathbf{Y}_2)}{\mathbf{Y}_\Delta} = \mathbf{4.04\ V\ \angle 145.03^\circ}$$

$$\mathbf{V}_3 = \frac{\mathbf{I}_2\ \left[(\mathbf{Y}_1+\mathbf{Y}_2)(\mathbf{Y}_3+\mathbf{Y}_4) - \mathbf{Y}_2^2\right] - \mathbf{Y}_2\mathbf{Y}_4\mathbf{I}_1}{\mathbf{Y}_\Delta} = \mathbf{25.94\ V\ \angle 78.07^\circ}$$

24.



$\mathbf{Y}_1 = \dfrac{1}{4\ \Omega\ \angle 0^\circ} = 0.25\ \text{S}\ \angle 0^\circ$

$\mathbf{Y}_2 = \dfrac{1}{6\ \Omega\ \angle 0^\circ} = 0.167\ \text{S}\ \angle 0^\circ$

$\mathbf{Y}_3 = \dfrac{1}{8\ \Omega\ \angle 0^\circ} = 0.125\ \text{S}\ \angle 0^\circ$

$\mathbf{Y}_4 = \dfrac{1}{2\ \Omega\ \angle -90^\circ} = 0.5\ \text{S}\ \angle 90^\circ$

$\mathbf{Y}_5 = \dfrac{1}{5\ \Omega\ \angle 90^\circ} = 0.2\ \text{S}\ \angle -90^\circ$

$\mathbf{Y}_6 = \dfrac{1}{4\ \Omega\ \angle 90^\circ} = 0.25\ \text{S}\ \angle -90^\circ$

$\mathbf{I}_1 = 4\ \text{A}\ \angle 0^\circ$

$\mathbf{I}_2 = 6\ \text{A}\ \angle 90^\circ$

$\mathbf{V}_1[\mathbf{Y}_1+\mathbf{Y}_2+\mathbf{Y}_3] - \mathbf{Y}_2\mathbf{V}_2 - \mathbf{Y}_3\mathbf{V}_3 = \mathbf{I}_1$
$\mathbf{V}_2[\mathbf{Y}_2+\mathbf{Y}_4+\mathbf{Y}_5] - \mathbf{Y}_2\mathbf{V}_1 - \mathbf{Y}_4\mathbf{V}_3 = 0$
$\mathbf{V}_3[\mathbf{Y}_3+\mathbf{Y}_4+\mathbf{Y}_6] - \mathbf{Y}_3\mathbf{V}_1 - \mathbf{Y}_4\mathbf{V}_2 = -\mathbf{I}_2$

$$\begin{aligned}
&[\mathbf{Y}_1+\mathbf{Y}_2+\mathbf{Y}_3]\mathbf{V}_1 - \mathbf{Y}_2\mathbf{V}_2 - \mathbf{Y}_3\mathbf{V}_3 = \mathbf{I}_1\\
&-\mathbf{Y}_2\mathbf{V}_1 + [\mathbf{Y}_2+\mathbf{Y}_4+\mathbf{Y}_5]\mathbf{V}_2 - \mathbf{Y}_4\mathbf{V}_3 = 0\\
&-\mathbf{Y}_3\mathbf{V}_1 - \mathbf{Y}_4\mathbf{V}_2 + [\mathbf{Y}_3+\mathbf{Y}_4+\mathbf{Y}_6]\mathbf{V}_3 = -\mathbf{I}_2
\end{aligned}$$

$$\mathbf{V}_1 = \frac{\mathbf{I}_1\left[(\mathbf{Y}_2+\mathbf{Y}_4+\mathbf{Y}_5)(\mathbf{Y}_3+\mathbf{Y}_4+\mathbf{Y}_6) - \mathbf{Y}_4^2\right] - \mathbf{I}_2\left[\mathbf{Y}_2\mathbf{Y}_4 + \mathbf{Y}_3(\mathbf{Y}_3+\mathbf{Y}_4+\mathbf{Y}_5)\right]}{\mathbf{Y}_\Delta = (\mathbf{Y}_1+\mathbf{Y}_2+\mathbf{Y}_3)\left[(\mathbf{Y}_2+\mathbf{Y}_4+\mathbf{Y}_5)(\mathbf{Y}_3+\mathbf{Y}_4+\mathbf{Y}_6) - \mathbf{Y}_4^2\right] - \mathbf{Y}_2\left[\mathbf{Y}_2(\mathbf{Y}_3+\mathbf{Y}_4+\mathbf{Y}_6)+\mathbf{Y}_3\mathbf{Y}_4\right] - \mathbf{Y}_3\left[\mathbf{Y}_2\mathbf{Y}_4+\mathbf{Y}_3(\mathbf{Y}_2+\mathbf{Y}_4+\mathbf{Y}_5)\right]}$$

$= \mathbf{15.13\ V\ \angle 1.29^\circ}$

$$\mathbf{V}_2 = \frac{\mathbf{I}_1\left[(\mathbf{Y}_2)(\mathbf{Y}_3+\mathbf{Y}_4+\mathbf{Y}_6)+\mathbf{Y}_3\mathbf{Y}_4\right]+\mathbf{I}_2\left[\mathbf{Y}_4(\mathbf{Y}_1+\mathbf{Y}_2+\mathbf{Y}_3) - \mathbf{Y}_2\mathbf{Y}_3\right]}{\mathbf{Y}_\Delta} = \mathbf{17.24\ V\ \angle 3.73^\circ}$$

$$\mathbf{V}_3 = \frac{\mathbf{I}_1\left[(\mathbf{Y}_3)(\mathbf{Y}_2+\mathbf{Y}_4+\mathbf{Y}_5) + \mathbf{Y}_2\mathbf{Y}_4\right] + \mathbf{I}_2\ \left[\mathbf{Y}_2^2 - (\mathbf{Y}_1+\mathbf{Y}_2+\mathbf{Y}_3)(\mathbf{Y}_2+\mathbf{Y}_4+\mathbf{Y}_5)\right]}{\mathbf{Y}_\Delta}$$

$= \mathbf{10.59\ V\ \angle -0.11^\circ}$

25. Left node: $\mathbf{V}_1$

$$\Sigma \mathbf{I}_i = \Sigma \mathbf{I}_o$$

$$4\mathbf{I}_x = \mathbf{I}_x + 5\ \text{mA}\ \angle 0^\circ + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2\ \text{k}\Omega}$$

Right node: $\mathbf{V}_2$

$$\Sigma \mathbf{I}_i = \Sigma \mathbf{I}_o$$

$$8\ \text{mA}\ \angle 0^\circ = \frac{\mathbf{V}_2}{1\ \text{k}\Omega} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{2\ \text{k}\Omega} + 4\mathbf{I}_x$$

Insert $\mathbf{I}_x = \dfrac{\mathbf{V}_1}{4\ \text{k}\Omega\ \angle -90^\circ}$

Rearrange, reduce and 2 equations with 2 unknowns result:

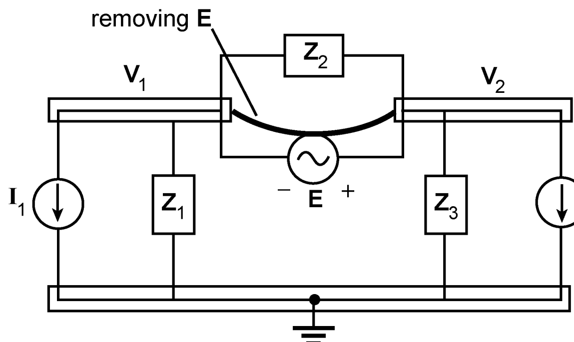
$\mathbf{V}_1[1.803 \angle 123.69°] + \mathbf{V}_2 = 10$

$\mathbf{V}_1[2.236 \angle 116.57°] + 3\,\mathbf{V}_2 = 16$

Determinants: $\mathbf{V}_1 = \mathbf{4.37\ V \angle -128.66°}$

$\mathbf{V}_2 = \mathbf{V}_{1k\Omega} = \mathbf{2.25\ V \angle 17.63°}$

26.



$\mathbf{Z}_1 = 1\text{ k}\Omega \angle 0°$
$\mathbf{Z}_2 = 2\text{ k}\Omega \angle 90°$
$\mathbf{Z}_3 = 3\text{ k}\Omega \angle -90°$
$\mathbf{I}_1 = 12\text{ mA} \angle 0°$
$\mathbf{I}_2 = 4\text{ mA} \angle 0°$
$\mathbf{E} = 10\text{ V} \angle 0°$

$$\Sigma \mathbf{I}_i = \Sigma \mathbf{I}_o$$

$$0 = \mathbf{I}_1 + \frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_2}{\mathbf{Z}_3} + \mathbf{I}_2$$

and $\dfrac{\mathbf{V}_1}{\mathbf{Z}_1} + \dfrac{\mathbf{V}_2}{\mathbf{Z}_3} = -\mathbf{I}_1 - \mathbf{I}_2$

with $\mathbf{V}_2 - \mathbf{V}_1 = \mathbf{E}$

Substituting and rearranging:

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_3}\right] = -\mathbf{I}_1 - \mathbf{I}_2 - \frac{\mathbf{E}}{\mathbf{Z}_3}$$

and solving for $\mathbf{V}_1$:

$\mathbf{V}_1 = \mathbf{15.4\ V \angle\ 178.2°}$

with $\mathbf{V}_2 = \mathbf{V}_C = \mathbf{5.41\ V \angle\ 174.87°}$

27. Left node: $\mathbf{V}_1$

$$\Sigma \mathbf{I}_i = \Sigma \mathbf{I}_o$$

$$2\text{ mA} \angle 0° = 12\text{ mA} \angle 0° + \frac{\mathbf{V}_1}{2\text{ k}\Omega} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1\text{ k}\Omega}$$

and $1.5\,\mathbf{V}_1 - \mathbf{V}_2 = -10$

Right node: $\mathbf{V}_2$

$$\Sigma \mathbf{I}_i = \Sigma \mathbf{I}_o$$

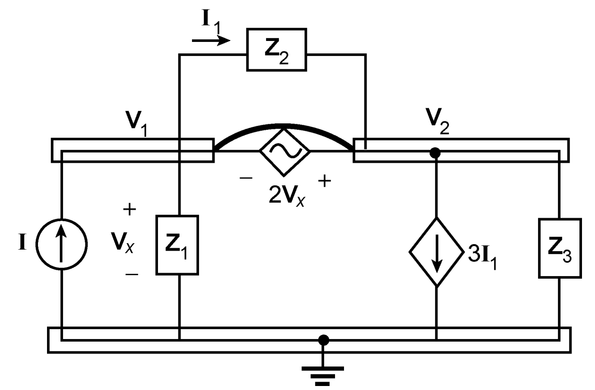
$$0 = 2\text{ mA} \angle 0° + \frac{\mathbf{V}_2 - \mathbf{V}_1}{1\text{ k}\Omega} - \frac{\mathbf{V}_2 - 6\,\mathbf{V}_x}{3.3\text{ k}\Omega}$$

and $2.7\,\mathbf{V}_1 - 3.7\,\mathbf{V}_2 = -6.6$

Using determinants: $\mathbf{V}_1 = \mathbf{V}_{2k\Omega} = \mathbf{-10.67\ V \angle 0° = 10.67\ V \angle 180°}$

$\mathbf{V}_2 = \mathbf{-6\ V \angle 0° = 6\ V \angle 180°}$

28.



$\mathbf{Z}_1 = 2\text{ k}\Omega \angle 0°$
$\mathbf{Z}_2 = 1\text{ k}\Omega \angle 0°$
$\mathbf{Z}_3 = 1\text{ k}\Omega \angle 0°$
$\mathbf{I} = 5\text{ mA} \angle 0°$

$\mathbf{V}_1$: $\mathbf{I} = \dfrac{\mathbf{V}_1}{\mathbf{Z}_1} + 3\mathbf{I}_1 + \dfrac{\mathbf{V}_2}{\mathbf{Z}_3}$

with $\mathbf{I}_1 = \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2}$

and $\mathbf{V}_2 - \mathbf{V}_1 = 2\mathbf{V}_x = 2\mathbf{V}_1$ or $\mathbf{V}_2 = 3\mathbf{V}_1$

Substituting will result in:

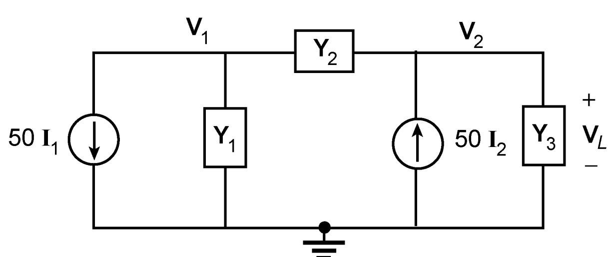
$$\mathbf{V}_1 \left[\frac{1}{\mathbf{Z}_1} + \frac{3}{\mathbf{Z}_2}\right] + 3\,\mathbf{V}_1 \left[\frac{1}{\mathbf{Z}_3} - \frac{3}{\mathbf{Z}_2}\right] = \mathbf{I}$$

or $$\mathbf{V}_1 \left[\frac{1}{\mathbf{Z}_1} - \frac{6}{\mathbf{Z}_2} + \frac{3}{\mathbf{Z}_3}\right] = \mathbf{I}$$

and $\mathbf{V}_1 = \mathbf{V}_x = \mathbf{-2\ V \angle 0°}$

with $\mathbf{V}_2 = \mathbf{-6\ V \angle 0°}$

29.



$\mathbf{I}_1 = \dfrac{E_i\angle\theta}{R_1\ \angle 0°} = 1 \times 10^{-3}\ \mathbf{E}_i$

$\mathbf{Y}_1 = \dfrac{1}{50\text{ k}\Omega} = 0.02\text{ mS} \angle 0°$

$\mathbf{Y}_2 = \dfrac{1}{1\text{ k}\Omega} = 1\text{ mS} \angle 0°$

$\mathbf{Y}_3 = 0.02\text{ mS} \angle 0°$

$\mathbf{I}_2 = (\mathbf{V}_1 - \mathbf{V}_2)\mathbf{Y}_2$

$\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{Y}_2\mathbf{V}_2 = -50\mathbf{I}_1$
$\mathbf{V}_2(\mathbf{Y}_2 + \mathbf{Y}_3) - \mathbf{Y}_2\mathbf{V}_1 = 50\mathbf{I}_2 = 50(\mathbf{V}_1 - \mathbf{V}_2)\mathbf{Y}_2 = 50\mathbf{Y}_2\mathbf{V}_1 - 50\mathbf{Y}_2\mathbf{V}_2$

$(\mathbf{Y}_1 + \mathbf{Y}_2)\mathbf{V}_1 - \mathbf{Y}_2\mathbf{V}_2 = -50\mathbf{I}_1$
$-51\mathbf{Y}_2\mathbf{V}_1 + (51\mathbf{Y}_2 + \mathbf{Y}_3)\mathbf{V}_2 = 0$

$$\mathbf{V}_L = \mathbf{V}_2 = \frac{-(50)(51)\mathbf{Y}_2\mathbf{I}_1}{(\mathbf{Y}_1 + \mathbf{Y}_2)(51\mathbf{Y}_2 + \mathbf{Y}_3) - 51\mathbf{Y}_2^2} = \mathbf{-2451.92\ E}_i$$

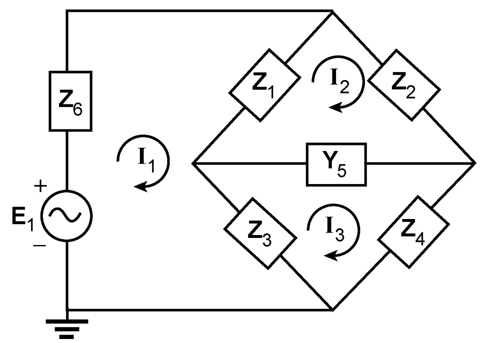
30. a. yes

$$\frac{\mathbf{Z}_1}{\mathbf{Z}_3} = \frac{\mathbf{Z}_2}{\mathbf{Z}_4}$$

$$\frac{5\times10^3\angle0^\circ}{2.5\times10^3\angle90^\circ} = \frac{8\times10^3\angle0^\circ}{4\times10^3\angle90^\circ}$$

$$2\angle-90^\circ = 2\angle-90^\circ \text{ (balanced)}$$

b. $\mathbf{Z}_1 = 5\text{ k}\Omega\angle0^\circ$, $\mathbf{Z}_2 = 8\text{ k}\Omega\angle0^\circ$
$\mathbf{Z}_3 = 2.5\text{ k}\Omega\angle90^\circ$, $\mathbf{Z}_4 = 4\text{ k}\Omega\angle90^\circ$
$\mathbf{Z}_5 = 5\text{ k}\Omega\angle-90^\circ$, $\mathbf{Z}_6 = 1\text{ k}\Omega\angle0^\circ$



$\mathbf{I}_1[\mathbf{Z}_1 + \mathbf{Z}_3 + \mathbf{Z}_6] - \mathbf{Z}_1\mathbf{I}_2 - \mathbf{Z}_3\mathbf{I}_3 = \mathbf{E}$
$\mathbf{I}_2[\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5] - \mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_5\mathbf{I}_3 = 0$
$\mathbf{I}_3[\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5] - \mathbf{Z}_3\mathbf{I}_1 - \mathbf{Z}_5\mathbf{I}_2 = 0$

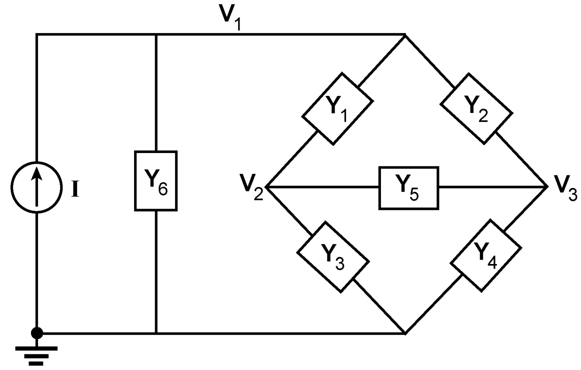
$$\begin{aligned}
&[\mathbf{Z}_1 + \mathbf{Z}_3 + \mathbf{Z}_6]\mathbf{I}_1 - \mathbf{Z}_1\mathbf{I}_2 - \mathbf{Z}_3\mathbf{I}_3 = \mathbf{E}\\
&-\mathbf{Z}_1\mathbf{I}_1 + [\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5]\mathbf{I}_2 - \mathbf{Z}_5\mathbf{I}_3 = 0\\
&-\mathbf{Z}_3\mathbf{I}_1 - \mathbf{Z}_5\mathbf{I}_2 + [\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5]\mathbf{I}_3 = 0
\end{aligned}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}[\mathbf{Z}_1(\mathbf{Z}_3+\mathbf{Z}_4+\mathbf{Z}_5) + \mathbf{Z}_3\mathbf{Z}_5]}{\mathbf{Z}_\Delta = (\mathbf{Z}_1+\mathbf{Z}_3+\mathbf{Z}_6)[(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_5)(\mathbf{Z}_3+\mathbf{Z}_4+\mathbf{Z}_5)-\mathbf{Z}_5^2]-\mathbf{Z}_1[\mathbf{Z}_1(\mathbf{Z}_3+\mathbf{Z}_4+\mathbf{Z}_5)-\mathbf{Z}_3\mathbf{Z}_5]-\mathbf{Z}_3[\mathbf{Z}_1\mathbf{Z}_5+\mathbf{Z}_3(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_5)]}$$

$$\mathbf{I}_3 = \frac{\mathbf{E}[\mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_5)]}{\mathbf{Z}_\Delta}$$

$$\mathbf{I}_{Z_5} = \mathbf{I}_2 - \mathbf{I}_3 = \frac{\mathbf{E}[\mathbf{Z}_1\mathbf{Z}_4 - \mathbf{Z}_3\mathbf{Z}_2]}{\mathbf{Z}\Delta} = \frac{\mathbf{E}[20\times10^6\angle90^\circ - 20\times10^6\angle90^\circ]}{\mathbf{Z}\Delta} = \mathbf{0\ A}$$

c.



$\mathbf{I} = \frac{\mathbf{E}_s}{\mathbf{R}_s} = \frac{10\text{ V}\angle 0°}{1\text{ k}\Omega\angle 0°} = 10\text{ mA }\angle 0°$

$\mathbf{Y}_1 = \frac{1}{5\text{ k}\Omega\angle 0°} = 0.2\text{ mS }\angle 0°$

$\mathbf{Y}_2 = \frac{1}{8\text{ k}\Omega\angle 0°} = 0.125\text{ mS }\angle 0°$

$\mathbf{Y}_3 = \frac{1}{2.5\text{ k}\Omega\angle 90°} = 0.4\text{ mS }\angle -90°$

$\mathbf{Y}_4 = \frac{1}{4\text{ k}\Omega\angle 90°} = 0.25\text{ mS }\angle -90°$

$\mathbf{Y}_5 = \frac{1}{5\text{ k}\Omega\angle -90°} = 0.2\text{ mS }\angle 90°$

$\mathbf{Y}_6 = \frac{1}{1\text{ k}\Omega\angle 0°}$

$\mathbf{V}_2 = 1\text{ mS }\angle 0°$

$$\mathbf{V}_1[\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_6] - \mathbf{Y}_1\mathbf{V}_2 - \mathbf{Y}_2\mathbf{V}_3 = \mathbf{I}$$
$$\mathbf{V}_2[\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5] - \mathbf{Y}_1\mathbf{V}_1 - \mathbf{Y}_5\mathbf{V}_3 = 0$$
$$\mathbf{V}_3[\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5] - \mathbf{Y}_2\mathbf{V}_1 - \mathbf{Y}_5\mathbf{V}_2 = 0$$

$$[\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_6]\mathbf{V}_1 - \mathbf{Y}_1\mathbf{V}_2 - \mathbf{Y}_2\mathbf{V}_3 = \mathbf{I}$$
$$-\mathbf{Y}_1\mathbf{V}_1 + [\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5]\mathbf{V}_2 - \mathbf{Y}_5\mathbf{V}_3 = 0$$
$$-\mathbf{Y}_2\mathbf{V}_1 - \mathbf{Y}_5\mathbf{V}_2 + [\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5]\mathbf{V}_3 = 0$$

$$\mathbf{V}_2 = \frac{\mathbf{I}[\mathbf{Y}_1(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) + \mathbf{Y}_2\mathbf{Y}_5]}{\mathbf{Y}_\Delta = (\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_6)[(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) - \mathbf{Y}_5^2] - \mathbf{Y}_1[\mathbf{Y}_1(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) + \mathbf{Y}_2\mathbf{Y}_5] - \mathbf{Y}_2[\mathbf{Y}_1\mathbf{Y}_5 + \mathbf{Y}_2(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)]}$$

$$\mathbf{V}_3 = \frac{\mathbf{I}[\mathbf{Y}_1\mathbf{Y}_5 + \mathbf{Y}_2(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)]}{\mathbf{Y}_\Delta}$$

$$\mathbf{V}_{Z_5} = \mathbf{V}_2 - \mathbf{V}_3 = \frac{\mathbf{I}[\mathbf{Y}_1\mathbf{Y}_4 - \mathbf{Y}_4\mathbf{Y}_3]}{\mathbf{Y}_\Delta} = \frac{\mathbf{I}\ [0.05\times 10^{-3}\angle -90° - 0.05\times 10^{-3}\angle -90°]}{\mathbf{Y}_\Delta}$$

$= \mathbf{0\ V}$

31. a.

$$\frac{\mathbf{Z}_1}{\mathbf{Z}_3} = \frac{\mathbf{Z}_2}{\mathbf{Z}_4}$$

$$\frac{4\times 10^3\angle 0°}{4\times 10^3\angle 90°} \stackrel{?}{=} \frac{4\times 10^3\angle 0°}{4\times 10^3\angle -90°}$$

$1\angle -90° \neq 1\angle 90°$ **(not balanced)**

b. The solution to 26(b) resulted in

$$\mathbf{I}_3 = \mathbf{I}_{X_C} = \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5))}{\mathbf{Z}\Delta}$$

where $\mathbf{Z}_\Delta = (\mathbf{Z}_1 + \mathbf{Z}_3 + \mathbf{Z}_6)[(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5)(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) - \mathbf{Z}_5^2] - \mathbf{Z}_1[\mathbf{Z}_1(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) - \mathbf{Z}_3\mathbf{Z}_5] - \mathbf{Z}_3[\mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5)]$

and $\mathbf{Z}_1 = 5\text{ k}\Omega\angle 0°$, $\mathbf{Z}_2 = 8\text{ k}\Omega\angle 0°$, $\mathbf{Z}_3 = 2.5\text{ k}\Omega\angle 90°$
$\mathbf{Z}_4 = 4\text{ k}\Omega\angle 90°$, $\mathbf{Z}_5 = 5\text{ k}\Omega\angle -90°$, $\mathbf{Z}_6 = 1\text{ k}\Omega\angle 0°$

and $\mathbf{I}_{X_C} = \mathbf{1.76\ mA\ \angle -71.54°}$

c. The solution to 26(c) resulted in

$$\mathbf{V}_3 = \mathbf{V}_{X_C} = \frac{\mathbf{I}[\mathbf{Y}_1\mathbf{Y}_5 + \mathbf{Y}_2(\mathbf{Y}_1+\mathbf{Y}_3+\mathbf{Y}_5)]}{\mathbf{Y}_\Delta}$$

where $\mathbf{Y}_\Delta = (\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_6)[(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) - \mathbf{Y}_5^2]$
$- \mathbf{Y}_1[\mathbf{Y}_1(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) + \mathbf{Y}_2\mathbf{Y}_5]$
$- \mathbf{Y}_2[\mathbf{Y}_1\mathbf{Y}_5 + \mathbf{Y}_2(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)]$

with $\mathbf{Y}_1 = 0.2\text{ mS}\angle 0°$, $\mathbf{Y}_2 = 0.125\text{ mS}\angle 0°$, $\mathbf{Y}_3 = 0.4\text{ mS}\angle -90°$
$\mathbf{Y}_4 = 0.25\text{ mS}\angle -90°$, $\mathbf{Y}_5 = 0.2\text{ mS}\angle 90°$

Source conversion: $\mathbf{Y}_6 = 1\text{ mS}\angle 0°$, $\mathbf{I} = 10\text{ mA}\angle 0°$
and $\mathbf{V}_3 = \mathbf{V}_{X_C} = \mathbf{7.03\ V\angle -18.46°}$

32. $\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_3\mathbf{Z}_2$

$(R_1 - jX_C)(R_x + jX_{L_x}) = R_3R_2$ $\qquad X_C = \frac{1}{\omega C} = \frac{1}{(10^3\text{ rad/s})(1\,\mu\text{F})} = 1\text{ k}\Omega$

$(1\text{ k}\Omega - j1\text{ k}\Omega)(R_x + jX_{L_x}) = (0.1\text{ k}\Omega)(0.1\text{ k}\Omega) = 10\text{ k}\Omega$

and $R_x + jX_{L_x} = \frac{10\times 10^3\ \Omega}{1\times 10^3 - j1\times 10^3} = \frac{10\times 10^3}{1.414\times 10^3\angle -45°} = 5\ \Omega + j5\ \Omega$

$\therefore R_x = \mathbf{5\ \Omega}$, $L_x = \frac{X_{L_x}}{\omega} = \frac{5\ \Omega}{10^3\text{ rad/s}} = \mathbf{5\ mH}$

33. $X_{C_1} = \frac{1}{\omega C_1} = \frac{1}{(1000\text{ rad/s})(3\,\mu\text{F})} = \frac{1}{3}\text{ k}\Omega$

$\mathbf{Z}_1 = R_1 \parallel X_{C_1}\angle -90° = (2\text{ k}\Omega\angle 0°) \parallel \frac{1}{3}\text{ k}\Omega\angle -90° = 328.8\ \Omega\angle -80.54°$

$\mathbf{Z}_2 = R_2\angle 0° = 0.5\text{ k}\Omega\angle 0°$, $\mathbf{Z}_3 = R_3\angle 0° = 4\text{ k}\Omega\angle 0°$
$\mathbf{Z}_4 = R_x + jX_{L_x} = 1\text{ k}\Omega + j6\text{ k}\Omega$

$$\frac{\mathbf{Z}_1}{\mathbf{Z}_3} = \frac{\mathbf{Z}_2}{\mathbf{Z}_4}$$

$$\frac{328.8\ \Omega\angle -80.54°}{4\text{ k}\Omega\angle 0°} \stackrel{?}{=} \frac{0.5\text{ k}\Omega\angle 0°}{6.083\ \Omega\angle 80.54°}$$

$82.2\times 10^{-3}\angle -80.54° \stackrel{\checkmark}{=} 82.2\times 10^{-3}\angle -80.54°$ **(balanced)**

34. Apply Eq. **18.6.**

35. For balance:

$R_1(R_x + j\,X_{L_x}) = R_2(R_3 + j\,X_{L_3})$

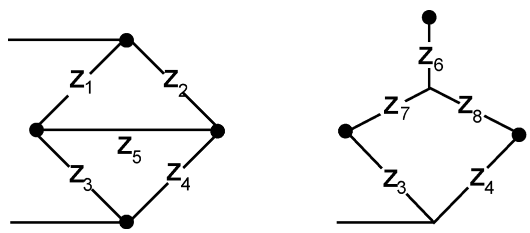
$R_1R_x + jR_1\,X_{L_x} = R_2R_3 + jR_2\,X_{L_3}$

$\therefore\ R_1R_x = R_2R_3$ and $R_x = \dfrac{\boldsymbol{R_2R_3}}{\boldsymbol{R_1}}$

$R_1\,X_{L_x} = R_2\,X_{L_3}$ and $R_1\omega L_x = R_2\omega L_3$

so that $L_x = \dfrac{\boldsymbol{R_2L_3}}{\boldsymbol{R_1}}$

36. a.



$\mathbf{Z}_1 = 8\ \Omega\ \angle{-90°} = -j8\ \Omega$
$\mathbf{Z}_2 = 4\ \Omega\ \angle 90° = +j4\ \Omega$
$\mathbf{Z}_3 = 8\ \Omega\ \angle 90° = +j8\ \Omega$
$\mathbf{Z}_4 = 6\ \Omega\ \angle{-90°} = -j6\ \Omega$
$\mathbf{Z}_5 = 5\ \Omega\ \angle 0°$

$$\mathbf{Z}_6 = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5} = 5\ \Omega\ \angle 38.66°$$

$$\mathbf{Z}_7 = \frac{\mathbf{Z}_1\mathbf{Z}_5}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5} = 6.25\ \Omega\ \angle{-51.34°}$$

$$\mathbf{Z}_8 = \frac{\mathbf{Z}_2\mathbf{Z}_5}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5} = 3.125\ \Omega\ \angle 128.66°$$

$\mathbf{Z}' = \mathbf{Z}_7 + \mathbf{Z}_3 = 3.9\ \Omega + j3.12\ \Omega = 4.99\ \Omega\ \angle 38.66°$

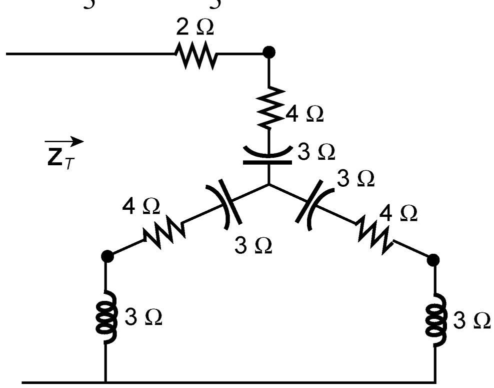
$\mathbf{Z}'' = \mathbf{Z}_8 + \mathbf{Z}_4 = -1.95\ \Omega - j3.56\ \Omega = 4.06\ \Omega\ \angle{-118.71°}$

$\mathbf{Z}' \parallel \mathbf{Z}'' = 10.13\ \Omega\ \angle{-67.33°} = 3.90\ \Omega - j9.35\ \Omega$

$\mathbf{Z}_T = \mathbf{Z}_6 + \mathbf{Z}' \parallel \mathbf{Z}'' = 7.80\ \Omega - j6.23\ \Omega = 9.98\ \Omega\ \angle{-38.61°}$

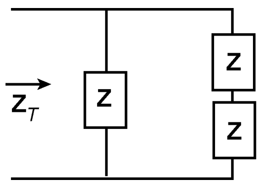
$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{120\text{ V}\ \angle 0°}{9.98\ \Omega\ \angle{-38.61°}} = \mathbf{12.02\ A\ \angle 38.61°}$$

37. $\mathbf{Z}_Y = \dfrac{\mathbf{Z}\Delta}{3} = \dfrac{12\ \Omega - j9\Omega}{3} = 4\ \Omega - j3\ \Omega$



$\mathbf{Z}_T = 2\ \Omega + 4\ \Omega + j3\ \Omega + [4\ \Omega - \cancel{j3\ \Omega} + \cancel{j3\ \Omega}] \parallel [4\ \Omega - \cancel{j3\ \Omega} + \cancel{j3\ \Omega}]$
$= 6\ \Omega - j3\ \Omega + 2\ \Omega$
$= 8\ \Omega - j3\ \Omega = 8.544\ \Omega \angle{-20.56°}$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{60\ \text{V} \angle 0°}{8.544\ \Omega \angle -20.56°} = \mathbf{7.02\ A \angle 20.56°}$$

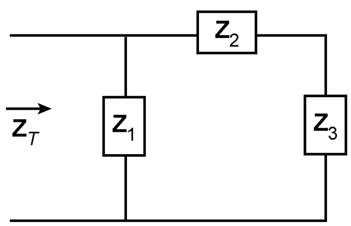
38. 

$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y = 3(3\ \Omega \angle 90°) = 9\ \Omega \angle 90°$
$\mathbf{Z} = 9\ \Omega \angle 90° \parallel (12\ \Omega - j16\ \Omega)$
$= 9\ \Omega \angle 90° \parallel 20\ \Omega \angle 53.13°$
$= 12.96\ \Omega \angle 67.13°$

$$\mathbf{Z}_T = \mathbf{Z} \parallel 2\mathbf{Z} = \frac{2\mathbf{Z}^2}{\mathbf{Z} + 2\mathbf{Z}} = \frac{2}{3}\mathbf{Z} = \frac{2}{3}\ [12.96\ \Omega \angle 67.13°] = 8.64\ \Omega \angle 67.13°$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\ \text{V} \angle 0°}{8.64\ \Omega \angle 67.13°} = \mathbf{11.57\ A \angle{-67.13°}}$$

39. $\mathbf{Z}_\Delta = 3\mathbf{Z}_Y = 3(5\ \Omega) = 15\ \Omega$



$\mathbf{Z}_1 = 15\ \Omega \angle 0° \parallel 5\ \Omega \angle{-90°}$
$= 4.74\ \Omega \angle{-71.57°}$
$\mathbf{Z}_2 = 15\ \Omega \angle 0° \parallel 5\ \Omega \angle{-90°} = 4.74\ \Omega \angle{-71.57°}$
$\mathbf{Z}_3 = \mathbf{Z}_1 = 4.74\ \Omega \angle{-71.57°}$
$= 1.5\ \Omega - j4.5\ \Omega$

$\mathbf{Z}_T =$ since $\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3$

$$\mathbf{Z}_T = \mathbf{Z}_1 \parallel (\mathbf{Z}_1 + \mathbf{Z}_1) = \mathbf{Z}_1 \parallel 2\mathbf{Z}_1 = \frac{2\mathbf{Z}_1^2}{\mathbf{Z}_1 + 2\mathbf{Z}_1} = \frac{2\mathbf{Z}_1^2}{3\mathbf{Z}_1} = \frac{2}{3}\mathbf{Z}_1$$

$$\mathbf{Z}_T = \frac{2}{3}(4.74\ \Omega \angle{-71.57°}) = 3.16\ \Omega \angle{-71.57°}$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\ \text{V} \angle 0°}{3.16\ \Omega \angle -71.57°} = \mathbf{31.65\ A \angle 71.57°}$$