Chapter 12

1. Φ: CGS: **5×10^4 Maxwells**, English: **5×10^4 lines**
   B: CGS: **8 Gauss**, English: **51.62 lines/in.^{2}**

2. Φ: SI **6×10^{-4} Wb**, English **60,000 lines**
   B: SI **0.465 T**, CGS **4.65×10^3 Gauss**, English **30,000 lines/in.^{2}**

3. a. $B = \dfrac{\Phi}{A} = \dfrac{4\times10^{-4}\text{ Wb}}{0.01\,\text{m}^2} = \mathbf{0.04\ T}$

4. a. $\mathscr{R} = \dfrac{l}{\mu A} = \dfrac{0.06\text{ m}}{\mu 2\times10^{-4}\text{ m}^2} = \dfrac{300}{\mu\,\text{m}}$

   b. $\mathscr{R} = \dfrac{l}{\mu A} = \dfrac{0.0762\text{ m}}{\mu 5\times10^{-4}\text{ m}^2} = \dfrac{152.4}{\mu\,\text{m}}$

   c. $\mathscr{R} = \dfrac{l}{\mu \text{A}} = \dfrac{0.1\text{ m}}{\mu 1\times10^{-4}\text{ m}^2} = \dfrac{1000}{\mu\,\text{m}}$

   from the above $\mathscr{R}_{(c)} > \mathscr{R}_{(a)} > \mathscr{R}_{(b)}$

5. $\mathscr{R} = \dfrac{\mathscr{F}}{\Phi} = \dfrac{400\text{ At}}{4.2\times10^{-4}\text{ Wb}} = \mathbf{952.4 \times 10^3\ At/Wb}$

6. $\mathscr{R} = \dfrac{\mathscr{F}}{\Phi} = \dfrac{120\text{ gilberts}}{72,000\text{ max wells}} = \mathbf{1.67 \times 10^{-3}\ rels\ (CGS)}$

7. $6\,\text{in.}\left[\dfrac{1\text{ m}}{39.37\text{ in.}}\right] = 0.1524\text{ m}$

   $H = \dfrac{\mathscr{F}}{l} = \dfrac{400\text{ At}}{0.1524\text{m}} = \mathbf{2624.67\ At/m}$

8. $\mu = \dfrac{2B}{H} = \dfrac{2(1200\times10^{-4}\text{ T})}{600\text{ At/m}} = \mathbf{4 \times 10^{-4}\ Wb/Am}$

9. $B = \dfrac{\Phi}{A} = \dfrac{10\times10^{-4}\text{ Wb}}{3\times10^{-3}\text{ m}^2} = 0.33\text{ T}$
   Fig. 12.7: $H \cong 800$ At/m
   $NI = Hl \Rightarrow I = Hl/N = (800\text{ At/m})(0.2\text{ m})/75\text{ t} = \mathbf{2.13\ A}$

10. $B = \dfrac{\Phi}{A} = \dfrac{3\times10^{-4}\text{ Wb}}{5\times10^{-4}\text{ m}^2} = 0.6\text{ T}$

Fig. 12.7, $H_{\text{iron}} = 2500$ At/m

Fig. 12.8, $H_{\text{steel}} = 70$ At/m

$NI = Hl_{(\text{iron})} + Hl_{(\text{steel})}$

$(100\text{ t})I = (H_{\text{iron}} + H_{\text{steel}})l$

$(100\text{ t})I = (2500\text{ At/m} + 70\text{ At/m})0.3\text{ m}$

$I = \dfrac{771\text{ A}}{100} = \mathbf{7.71\text{ A}}$

11. a. $N_1I_1 + N_2I_2 = Hl$

$B = \dfrac{\Phi}{A} = \dfrac{12\times10^{-4}\text{ Wb}}{12\times10^{-4}\text{ m}^2} = 1\text{ T}$

Fig. 12.7: $H \cong 750$ At/m

$N_1(2\text{ A}) + 30\text{ At} = (750\text{ At/m})(0.2\text{ m})$

$N_1 = \mathbf{60\text{ t}}$

b. $\mu = \dfrac{B}{H} = \dfrac{1\text{ T}}{750\text{ At/m}} = \mathbf{13.34 \times 10^{-4}\text{ Wb/Am}}$

12. a. $80{,}000\text{ }\cancel{\text{lines}}\left[\dfrac{1\text{ Wb}}{10^8\text{ }\cancel{\text{lines}}}\right] = 8\times10^4\times10^{-8}\text{ Wb} = 8\times10^{-4}\text{ Wb}$

$l_{(\text{cast steel})} = 5.5\text{ }\cancel{\text{in.}}\left[\dfrac{1\text{ m}}{39.37\text{ }\cancel{\text{in.}}}\right] = 0.14\text{ m}$

$l_{(\text{sheet steel})} = 0.5\text{ }\cancel{\text{in.}}\left[\dfrac{1\text{ m}}{39.37\text{ }\cancel{\text{in.}}}\right] = 0.013\text{ m}$

$\text{Area} = 1\text{ }\cancel{\text{in.}}^2\left[\dfrac{1\text{ m}}{39.37\text{ }\cancel{\text{in.}}}\right]\left[\dfrac{1\text{ m}}{39.37\text{ }\cancel{\text{in.}}}\right] = 6.45\times10^{-4}\text{ m}^2$

$B = \dfrac{\Phi}{A} = \dfrac{8\times10^{-4}\text{ Wb}}{6.45\times10^{-4}\text{ m}^2} = 1.24\text{ T}$

Fig 12.8: $H_{\text{sheet steel}} \cong 460$ At/m, Fig. 12.7: $H_{\text{cast steel}} \cong 1275$ At/m

$NI = Hl_{(\text{sheet steel})} + Hl_{(\text{cast iron})}$

$= (460\text{ At/m})(0.013\text{ m}) + (1275\text{ At/m})(0.14\text{ m})$

$= 5.98\text{ At} + 178.50\text{ At}$

$NI = \mathbf{184.48\text{ At}}$

b. Cast steel: $\mu = \dfrac{B}{H} = \dfrac{1.24\text{ T}}{1275\text{ At/m}} = \mathbf{9.73\times10^{-4}\text{ Wb/Am}}$

Sheet steel: $\mu = \dfrac{B}{H} = \dfrac{1.24\text{ T}}{460\text{ At/m}} = \mathbf{26.96\times10^{-4}\text{ Wb/Am}}$

13. $N_1I + N_2 = \underbrace{Hl}_{\text{cast steel}} + \underbrace{Hl}_{\text{cast iron}}$

$(20\text{ t})I + (30\text{ t})I =$ "

$(50\text{ t})I =$ "

$$B = \frac{\Phi}{A} \text{ with } 0.25 \text{ in.}^2 \left[\frac{1\text{ m}}{39.37\text{ in.}}\right]\left[\frac{1\text{ m}}{39.37\text{ in.}}\right] = 1.6 \times 10^{-4}\text{ m}^2$$

$$B = \frac{0.8\times10^{-4}\text{ Wb}}{1.6\times10^{-4}\text{ m}^2} = 0.5\text{ T}$$

Fig. 12.8: $H_{\text{cast steel}} \cong 280$ At/m

Fig. 12.7: $H_{\text{cast iron}} \cong 1500$ At/m

$$l_{\text{cast steel}} = 5.5\text{ in.}\left[\frac{1\text{ m}}{39.37\text{ in.}}\right] = 0.14\text{ m}$$

$$l_{\text{cast iron}} = 2.5\text{ in.}\left[\frac{1\text{ m}}{39.37\text{ in.}}\right] = 0.064\text{ m}$$

$(50\text{ t})I = (280\text{ At/m})(0.14\text{ m}) + (1500\text{ At/m})(0.064\text{ m})$

$50I = 39.20 + 96.00 = 135.20$

$I =$ **2.70 A**

14. a. $l_{ab} = l_{ef} = 0.05$ m, $l_{af} = 0.02$ m, $l_{bc} = l_{de} = 0.0085$ m

$NI = 2H_{ab}l_{ab} + 2H_{bc}l_{bc} + H_{fa}l_{fa} + H_gl_g$

$$B = \frac{\Phi}{A} = \frac{2.4\times10^{-4}\text{ Wb}}{2\times10^{-4}\text{ m}^2} = 1.2\text{ T} \Rightarrow H \cong 360\text{ At/m (Fig. 12.8)}$$

$100I = 2(360\text{ At/m})(0.05\text{ m}) + 2(360\text{ At/m})(0.0085\text{ m})$

$+ (360\text{ At/m})(0.02\text{ m}) + 7.97 \times 10^5(1.2\text{ T})(0.003\text{ m})$

$= 36\text{ At} + 6.12\text{ At} + 7.2\text{ At} + 2869\text{ At}$

$100I = 2918.32$ At

$I \cong$ **29.18 A**

b. air gap: metal = 2869 At**:**49.72 At = **58.17:1**

$\mu_{\text{sheet steel}} = \frac{B}{H} = \frac{1.2\text{ T}}{360\text{ At/m}} = \mathbf{3.33 \times 10^{-3}\text{ Wb/Am}}$

$\mu_{\text{air}} = \mathbf{4\pi \times 10^{-7}\text{ Wb/Am}}$

$\mu_{\text{sheet steel}}\text{:}\ \mu_{\text{air}} = 3.33 \times 10^{-3}\text{ Wb/Am}:4\pi \times 10^{-7} \cong$ **2627:1**

15. $4\text{ cm}\left[\frac{1\text{ m}}{100\text{ cm}}\right] = 0.04\text{ m}$

$$f = \frac{1}{2}NI\frac{d\phi}{dx} = \frac{1}{2}(80\text{ t})(0.9\text{ A})\frac{(8\times10^{-4}\text{ Wb} - 0.5\times10^{-4}\text{ Wb})}{\frac{1}{2}(0.04\text{ m})} = \frac{36(7.5\times10^{-4})}{0.02}$$

$=$ **1.35 N**

16. $C = 2\pi r = (6.28)(0.3 \text{ m}) = 1.88 \text{ m}$

$$B = \frac{\Phi}{A} = \frac{2\times10^{-4}\text{ Wb}}{1.3\times10^{-4}\text{ m}^2} = 1.54 \text{ T}$$

Fig. 12.7: $H_{\text{sheet steel}} \cong 2100$ At/m

$H_g = 7.97 \times 10^5 B_g = (7.97 \times 10^5)(1.54 \text{ T}) = 1.23 \times 10^6$ At/m

$N_1 I_1 + N_2 I_2 = H_g l_g + Hl_{(\text{sheet steel})}$

$(200 \text{ t})I_1 + (40 \text{ t})(0.3 \text{ A}) = (1.23 \times 10^6 \text{ At/m})(2 \text{ mm}) + (2100 \text{ At/m})(1.88 \text{ m})$

$I_1 = \mathbf{31.98\ A}$

17. a. $0.2 \cancel{\text{cm}} \left[\frac{1\text{ m}}{100\cancel{\text{ cm}}}\right] = 2 \times 10^{-3}$ m

$$A = \frac{\pi d^2}{4} = \frac{(3.14)(0.01\text{ m})^2}{4} = 0.79 \times 10^{-4} \text{ m}^2$$

$NI = H_g l_g$, $H_g = 7.96 \times 10^5 B_g$

$$(200 \text{ t})I = \left[(7.96\times10^5)\left(\frac{0.2\times10^{-4}\text{ Wb}}{0.79\times10^{-4}\text{ m}^2}\right)\right]2 \times 10^{-3} \text{ m}$$

$I = \mathbf{2.02\ A}$

b. $$B_g = \frac{\Phi}{A} = \frac{2\times10^{-4}\text{ Wb}}{0.79\times10^{-4}\text{ m}^2} = 0.25 \text{ T}$$

$$F \cong \frac{1}{2}\frac{B_g^2 A}{\mu_o} = \frac{1}{2}\frac{(0.25\text{ T})^2(0.79\times10^{-4}\text{ m}^2)}{4\pi\times10^{-7}}$$

$\cong$ **2 N**

18. **Table:**

Section	Φ(Wb)	A(m²)	*B*(T)	*H*	*l*(m)	*Hl*
a–b, g–h		5×10^{-4}			0.2	
b–c, f–g	2×10^{-4}	5×10^{-4}			0.1	
c–d, e–f	2×10^{-4}	5×10^{-4}			0.099	
a–h		5×10^{-4}			0.2	
b–g		2×10^{-4}			0.2	
d–e	2×10^{-4}	5×10^{-4}			0.002	

$$B_{bc} = B_{cd} = B_g = B_{ef} = B_{fg} = \frac{\Phi}{A} = \frac{2\times10^{-4}\ \text{Wb}}{5\times10^{-4}\ \text{m}^2} = 0.4\ \text{T}$$

Air gap: $H_g = 7.97 \times 10^5(0.4\ \text{T}) = 3.19 \times 10^5$ At/m

$H_g l_g = (3.19 \times 10^5\ \text{At/m})(2\ \text{mm}) = 638$ At

Fig 12.8: $H_{bc} = H_{cd} = H_{ef} = H_{fg} = 55$ At/m

$H_{bc}l_{bc} = H_{fg}l_{fg} = (55\ \text{At/m})(0.1\ \text{m}) = 5.5$ At

$H_{cd}l_{cd} = H_{ef}l_{ef} = (55\ \text{At/m})(0.099\ \text{m}) = 5.45$ At

For loop 2: $\Sigma\mathscr{F} = 0$

$H_{bc}l_{bc} + H_{cd}l_{cd} + H_g l_g + H_{ef}l_{ef} + H_{fg}l_{fg} - H_{gb}l_{gb} = 0$

$5.5\ \text{At} + 5.45\ \text{At} + 638\ \text{At} + 5.45\ \text{At} + 5.50\ \text{At} - H_{gb}l_{gb} = 0$

$H_{gb}l_{gb} = 659.90$ At

and $H_{gb} = \dfrac{659.90\ \text{At}}{0.2\ \text{m}} = 3300$ At/m

Fig 12.7: $B_{gb} \cong 1.55$ T

with $\Phi_2 = B_{gb}A = (1.55\ \text{T})(2 \times 10^{-4}\ \text{m}^2) = 3.1 \times 10^{-4}$ Wb

$\Phi_T = \Phi_1 + \Phi_2$

$= 2 \times 10^{-4}\ \text{Wb} + 3.1 \times 10^{-4}$ Wb

$= 5.1 \times 10^{-4}\ \text{Wb} = \Phi_{ab} = \Phi_{ha} = \Phi_{gh}$

$$B_{ab} = B_{ha} = B_{gh} = \frac{\Phi_T}{A} = \frac{5.1\times10^{-4}\ \text{Wb}}{5\times10^{-4}\ \text{m}^2} = 1.02\ \text{T}$$

B–H curve: (Fig 12.8):

$H_{ab} = H_{ha} = H_{gh} \cong 180$ At/m

$H_{ab}l_{ab} = (180\ \text{At/m})(0.2\ \text{m}) = 36$ At

$H_{ha}l_{ha} = (180\ \text{At/m})(0.2\ \text{m}) = 36$ At

$H_{gh}l_{gh} = (180\ \text{At/m})(0.2\ \text{m}) = 36$ At

which **completes** the table!

Loop #1: $\Sigma\mathscr{F} = 0$

$NI = H_{ab}l_{ab} + H_{bg}l_{bg} + H_{gh}l_{gh} + H_{ah}l_{ah}$

$(200\ \text{t})I = 36\ \text{At} + 659.49\ \text{At} + 36\ \text{At} + 36$ At

$(200\ \text{t})I = 767.49$ At

$I \cong$ **3.84 A**

19. $NI = Hl$

$l = 2\pi r = (6.28)(0.08\ \text{m}) = 0.50$ m

$(100\ \text{t})(2\ \text{A}) = H(0.50\ \text{m})$

$H = 400$ At/m

Fig. 12.8: $B \cong 0.68$ T

$\Phi = BA = (0.68\ \text{T})(0.009\ \text{m}^2)$

$\Phi =$ **6.12 mWb**

20. $NI = H_{ab}(l_{ab} + l_{bc} + l_{de} + l_{ef} + l_{fa}) + H_g l_g$

$300 \text{ At} = H_{ab}(0.8 \text{ m}) + 7.97 \times 10^5 B_g(0.8 \text{ mm})$

$300 \text{ At} = H_{ab}(0.8 \text{ m}) + 637.6\, B_g$

Assuming $637.6\, B_g \gg H_{ab}(0.8 \text{ m})$

then $300 \text{ At} = 637.6\, B_g$

and $B_g = 0.47 \text{ T}$

$\Phi = BA = (0.47 \text{ T})(2 \times 10^{-4} \text{ m}^2) = 0.94 \times 10^{-4} \text{ Wb}$

$B_{ab} = B_g = 0.47 \text{ T} \Rightarrow H \cong 270 \text{ At/m}$ (Fig. 12.8)

$300 \text{ At} = (270 \text{ At/m})(0.8 \text{ m}) + 637.6(0.47 \text{ T})$

$300 \text{ At} \neq 515.67 \text{ At}$

$\therefore$ Poor approximation!

$$\frac{300 \text{ At}}{515.67 \text{ At}} \times 100\% \cong 58\%$$

Reduce Φ to 58%

$0.58(0.94 \times 10^{-4} \text{ Wb}) = 0.55 \times 10^{-4} \text{ Wb}$

$$B = \frac{\Phi}{A} = \frac{0.55 \times 10^{-4} \text{ Wb}}{2 \times 10^{-4} \text{ m}^2} = 0.28 \text{ T} \Rightarrow H \cong 190 \text{ At/m} \text{ (Fig. 12.8)}$$

$300 \text{ At} = (190 \text{ At/m})(0.8 \text{ m}) + 637.6(0.28 \text{ T})$

$300 \text{ At} \neq 330.53 \text{ At}$

Reduce Φ another $10\% = 0.55 \times 10^{-4} \text{ Wb} - 0.1(0.55 \times 10^{-4} \text{ Wb})$

$= 0.495 \times 10^{-4} \text{ Wb}$

$$B = \frac{\Phi}{A} = \frac{0.495 \times 10^{-4} \text{ Wb}}{2 \times 10^{-4} \text{ m}^2} = 0.25 \text{ T} \Rightarrow H \cong 175 \text{ At/m} \text{ (Fig. 12.7)}$$

$300 \text{ At} = (175 \text{ At/m})(0.8) + 637.6(0.28 \text{ T})$

$300 \text{ At} \neq 318.53 \text{ At}$ but within 5% $\therefore$ OK

$\Phi \cong \mathbf{0.55 \times 10^{-4} \text{ Wb}}$

21. a. $1\tau = 0.632\, T_{max}$

$T_{max} \cong 1.5 \text{ T}$ for cast steel

$0.632(1.5 \text{ T}) = 0.945 \text{ T}$

At 0.945 T, $H \cong 700 \text{ At/m}$ (Fig. 12.7)

$\therefore \boldsymbol{B = 1.5 \text{ T}(1 - e^{-H/700 \text{ At/m}})}$

b. $H = 900 \text{ At/m}$:

$$B = 1.5 \text{ T}\left(1 - e^{-\frac{900 \text{ At/m}}{700 \text{ At/m}}}\right) = \mathbf{1.09 \text{ T}}$$

Graph: $\cong$ **1.1 T**

$H = 1800 \text{ At/m}$:

$$B = 1.5 \text{ T}\left(1 - e^{-\frac{1800 \text{ At/m}}{700 \text{ At/m}}}\right) = \mathbf{1.39 \text{ T}}$$

Graph: $\cong$ **1.38 T**

$H = 2700 \text{ At/m}$:

$$B = 1.5\left(1 - e^{-\frac{2700 \text{ At/m}}{700 \text{ At/m}}}\right) = \mathbf{1.47 \text{ T}}$$

Graph: $\cong$ **1.47 T**

Excellent comparison!

c. $B = 1.5\text{ T}(1 - e^{-H/700\text{ At/m}}) = 1.5\text{ T} - 1.5\text{ T}e^{-H/700\text{ At/m}}$

$B - 1.5\text{ T} = -1.5\text{ T}e^{-H/700\text{ At/m}}$

$1.5 - B = 1.5\text{ T}e^{-H/700\text{ At/m}}$

$$\frac{1.5\text{ T} - B}{1.5\text{ T}} = e^{-H/700\text{ At/m}}$$

$$\log_e\left(1 - \frac{B}{1.5\text{ T}}\right) = \frac{-H}{700\text{ At/m}}$$

and $H = -700\log_e\left(1 - \frac{B}{1.5\text{ T}}\right)$

d. $B = 1$ T:

$$H = -700\log_e\left(1 - \frac{1\text{ T}}{1.5\text{ T}}\right) = \mathbf{769.03\ At/m}$$

Graph: $\cong$ **750 At/m**

$B = 1.4$ T:

$$H = -700\log_e\left(1 - \frac{1.4\text{ T}}{1.5\text{ T}}\right) = \mathbf{1895.64\ At/m}$$

Graph: $\cong$ **1920 At/m**

e. $H = -700\log_e\left(1 - \frac{B}{1.5\text{ T}}\right)$

$= -700\log_e\left(1 - \frac{0.2\text{ T}}{1.5\text{ T}}\right)$

$= 100.2$ At/m

$$I = \frac{Hl}{N} = \frac{(100.2\text{ At/m})(0.16\text{ m})}{400\text{ t}} = \mathbf{40.1\ mA}$$

vs 44 mA for Ex. 12.1